

Calendars

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15.1 Introduction

15.1.1 Overview

A calendar is a system for assigning a unique name, its date, to every day so that the order of the dates is apparent and the interval in days between two dates can be readily calculated. A date is generally specified by three numbers: a *day* of the month, *D*, a *month*, *M* and a *year*, *Y*; in most calendars, the months also have names.

Most calendars are based on astronomical cycles so that the days of the month cycle in approximate synchrony with the lunations (the cycle of the phases of the Moon) and the months cycle in approximate synchrony with the seasons of the year.

Since the number of lunations in a year and the number of days in a lunation are not integral (and also slowly change with time), extra intercalary days or months may be inserted into occasional years to maintain a long-term average synchrony with the astronomical cycles. Years in which a day or month is intercalated are sometime called *leap years*. The Romans called years with an extra day *bissextile years*.

Solar calendars, such as the current *Gregorian calendar*, maintain accurate synchrony with the astronomical year while abandoning any attempt to keep the months in synchrony with the lunations of the Moon. *Lunar calendars*, such as the Islamic calendar, maintain synchrony of the months and abandon synchrony with the Sun. *Luni-solar calendars*, such as the Jewish, Chinese and Christian Ecclesiastic calendars, attempt to maintain synchrony with both.

According to a recent estimate (Fraser 1987), there are about forty calendars in use in the world today and more that have been abandoned. This chapter is concerned with only a few of these. We discuss here nine. The first of these is the Gregorian calendar, which is now used throughout the world for secular purposes. It is the official calendar of the United Kingdom (since 1752), but not of the United States (which has no official calendar). We discuss a further six (the Julian, Jewish, Islamic, Indian, Chinese and Bahá'í calendars), which are in current use to determine the proper dates of religious or cultural activities. We discuss two more (The Ancient Egyptian and the French Republican calendars), which are of historical interest. Brief historical summaries and the details of the operation are given for each of these nine calendars, but we do not discuss their usage. Algorithms are given for arithmetic calendars (see § 15.1.4) for converting dates in one to those in others.

Despite a vast literature on calendars, truly authoritative references, particularly in English, are difficult to find. One such is Reingold and Dershowitz (1997, 2001). Richards (1998)

surveys a broad variety of calendars and Aveni (1990) stresses their cultural contexts rather than their operational details. Fotheringham (1935), Doggett (1992) and the *Encyclopaedia of Religion and Ethics* (Hastings 1910), in its section on “Calendars,” offer basic information on historical calendars. Blackburn and Holford-Stevens (1999) give detailed information about customs appertaining to each day of the year and on a variety of calendars. The sections on “Calendars” and “Chronology” in all editions of the *Encyclopaedia Britannica* provide useful historical surveys. Ginzel (1906,1911) remains an authoritative, if dated, standard of calendrical scholarship. References on individual calendars are given in the relevant sections. See also URL[1], URL[2], URL[3], URL[4], and URL[5].

15.1.2 Uses of Calendars

Calendars have been used from the earliest times to regulate hunting, agricultural practices and other economic activities. Later they were important in regulating religious rituals, fasts and feasts. Most major religions use a special calendar and most calendars are associated with a religion. From the beginnings of civilization calendars have been used to date receipts, contracts and other documents and to order social life and to define holidays.

Details of their uses in religious practices are discussed in the *Encyclopaedia of Religion and Ethics* (Hastings 1910) and in Blackburn and Holford-Stevens (1999).

15.1.3 Astronomical Bases of Calendars

The principal astronomical cycles are the year (based on the revolution of the Earth around the Sun), the month (based on the revolution of the Moon around the Earth) and the day (based on the rotation of the Earth on its axis). The complexity of calendars arises because the month and the year do not comprise an integral number of days, and because they are neither constant nor perfectly commensurable with each other (see Doggett 1992).

The *tropical year* is today defined as the time needed for the Sun’s mean longitude to increase by 360° (Danjon 1959; Meeus and Savoie 1992). This varies from year to year by several minutes, but it may be averaged over several years to give the mean tropical year. It may be noted that this definition differs from the traditional definition, which is the mean period between two vernal equinoxes.

The intervals between any particular pair of *equinoxes* or *solstices* are not equal to one another or to the tropical year; they are also subject to variations from year to year but may be averaged over a number of years. The arithmetic mean of the four average intervals based on the two equinoxes and the two solstices is equivalent to the value of the mean tropical year. These matters are discussed by Steel (2000).

It is sometimes assumed that the tropical year represents the period of the cycle of the seasons. It may be noted, however, that the Gregorian calendar is tied to the vernal equinox year and the Chinese calendar to the winter solstice.

The following approximate expression, based on the *orbital elements* of Laskar (1986), may be used to calculate the length of the mean tropical year in the distant past. Note, however, that *The Astronomical Almanac* has not used these equations, nor does it use the orbital elements from Laskar, but starting from the 2004 edition, it uses the orbital elements of Simon et al. (1994):

$$365^{\text{d}}.242\ 189\ 6698 - 0.000\ 006\ 153\ 59\ T - 7.29 \times 10^{-10}\ T^2 + 2.64 \times 10^{-10}\ T^3. \quad (15.1)$$

Table 15.1 Approximate lengths of the astronomical cycles

Year ^a	Days in Year ^b 1 day = 86400 seconds (SI)	Days in lunation ^c	Length of day	Days ^d /year	Days ^e /lunation
2000	365.24219	29.53059	86400.003	365.24217	29.53059
1500	365.24222	29.53059	86399.995	365.24224	29.53059
1000	365.24225	29.53059	86399.886	365.24231	29.53059
500	365.24228	29.53059	86399.878	365.24237	29.53059
0	365.24231	29.53058	86399.969	365.24244	29.53059
-500	365.24234	29.53058	86399.961	365.24250	29.53060
rate of change ^f (ppb)	-17	73	20	-37	53

^a Astronomical year; i.e., the year Y B.C. is designated astronomical year 1–Y.

^b The mean tropical year as given by Eq. 15.1.

^c Mean Synodic period as given by Eq. 15.2

^d Days of a length in seconds (SI) as given in column 4

^e Days of a length in seconds (SI) as given in column 4

^f The rates of change of these periods themselves change over the centuries, but the proportional rates of increase, expressed as parts per billion (10^9) per century, are given for the year 2000

The synodic month, or lunation, the mean interval between conjunctions of the Moon and Sun, corresponds to the cycle of lunar phases. The following expression for the synodic month is based on the lunar theory of Chapront-Touzé and Chapront (1988):

$$29^d:530\ 588\ 8531 + 0.000\ 000\ 216\ 21\ T - 3.64 \times 10^{-10} T^2 \quad (15.2)$$

but any particular cycle may vary from the mean by up to seven hours.

In these expressions, T is the number of *Julian centuries* (of 36525 days) measured from 2000 January 1 in *Terrestrial Time* (TT). It is given by $T = (J - 245\ 1545.0)/36525$ where J is the *Julian date* (see § 15.1.10).

The lengths of the tropical year and *synodic month* are given by these formulae in days of 86400 seconds (SI) of *International Atomic Time* (TAI). See Chapter 3 for further information on timescales.

Historians (Stephenson 1997) have also found that the length of the mean solar day expressed in TAI is not constant. It appears to be increasing by about 0.0017 seconds per century because of tidal and other effects. It also suffers from both periodic and small random changes.

The lengths of the cycles change slowly with time and expressions 15.1 and 15.2 only represent approximations and are included in this chapter only for illustrative reasons. The lengths of these cycles calculated from them for representative years are shown in Table 15.1.

15.1.4 Astronomical, Observational and Arithmetic Calendars

Some calendars, termed observational calendars, rely on actual sightings of a new moon, solstice, equinox; or some other natural phenomenon to determine the start of the months and the years or when to intercalate a day or a month. The times of the sightings are subject

to error on account of the weather and such calendars cannot be constructed in advance. Examples are the ancient Jewish calendar and the Islamic calendar as it is generally used to determine important religious dates (such as the start of Ramadan); both of these depend on sightings of the crescent of the new moon. The heliacal rising of Sopdet (Sirius) may, at one time, have signalled the start of the ancient Egyptian year. It is likely that many calendars now lost were observational. Note that the local time at which an observation is made may fall in different days according to the locality of the observer.

Astronomical calendars depend on a model to predict the times of such events. These models have changed over the centuries as astronomical knowledge improved. Examples of astronomical calendars include the Chinese and Indian luni-solar calendars, the original form of the French Republican calendar and one form of the Bahá'í calendar. Astronomical calendars usually specify that the events be observed or calculated for a particular meridian. Thus events used in the Chinese calendar are observed at a meridian at 120° East (but before 1929 at the meridian of Beijing). The French Republican calendar specified the meridian of Paris.

Arithmetic calendars abandon any explicit references to astronomical events and are regulated by fixed rules, though they may have been designed in an attempt to keep their years or months in step with the astronomical periods. Examples include the current Gregorian calendar and its predecessor, the Julian calendar, as well as the ancient Egyptian calendar, the modern Jewish calendar and the civil Islamic calendar (used by historians of Islamic culture).

15.1.5 Visibility of the Crescent Moon

Several observational calendars rely on the first sighting of new moons. Under optimal conditions, the crescent Moon has been sighted about 15.4 hours after the astronomical new moon (i.e., the *conjunction*) (Schaefer 1988). Usually, however, it is not seen until it is more than twenty-four hours old. Babylonian astronomers were the first to develop methods for calculating first visibility, though no surviving tables are explicitly concerned with this (Neugebauer 1975, pp. 533–540). The earliest known visibility tables are by al-Khwarizmi, a ninth-century astronomer of Baghdad (King 1987). These tables, and many subsequent tables, were based on the Indian criterion that the Moon will be visible if the local hour angle of the Moon at sunset is equal to or less than 78°. With the development of Islamic astronomy more complex criteria were developed.

Modern models for predicting first visibility incorporate celestial mechanics, spherical astronomy, selenology, atmospheric physics, and ophthalmology. Bruin (1977) was the first to prepare such a model. Ilyas (1984), recognising that the Islamic calendar is used around the world, introduced the concept of an *International Lunar Dateline*, west of which the Moon should be visible under good observing conditions. Schaefer (1988) has further investigated the theory of this. Extensive observing programs were once organised by Doggett, Seidelmann, and Schaefer (1988) and Doggett and Schaefer (1989); see also URL[7].

15.1.6 Non-astronomical Cycles and the Week

The calendars of many societies have incorporated cycles with no astronomical basis. Cycles of days include the eight-day cycle (*Nundinae*) of the Romans, the 13 and 20 day cycles of Central America and the market weeks, of various lengths, of Africa. The Ancient Egyptian

calendar and the French Republican calendar used a 10-day cycle, and the Bolsheviks in Russia experimented with a four-day “week.” Most of the world today recognises the seven-day week which has religious significance for Christians, Jews and Muslims.

The seven-day week probably originated in Babylon as a useful, if inexact, division of the lunar month and was probably adopted by the Jewish people at the time of the Babylonian exile in the sixth century B.C.. It is likely that it has run continuously and without interruption since these times to the present day (but see Zerubavel E. (1989). The seven-day cycle was widely used in the Roman Empire from the first century B.C.. According to Cassius Deo each of the seven “wandering stars,” the Sun, the Moon and the five naked-eye planets known to the ancients, was assigned cyclically to each of the 24 hours of each day in the order of their apparent speed in the heavens. The star assigned to the first hour was the regent of the day. The names of the regents gave each day its name and the order of the days in this astrological week. These names, or their equivalents in other languages and pantheons, are still used today. Saturn’s day coincided with the Jewish Sabbath, though the Jews never used the Roman names.

Colson (1926) and Zerubavel (1989) are standard references on the week. Richards (1998) gives a comprehensive list of the names of the days of the week. Zerubavel (1989) also considers other week-like numerical cycles used by past and present cultures.

Cycles of years have also had significance for various cultures. The Romans used a 15-year cycle of indiction for taxation purposes. The Chinese have a cycle of 60 years comprised of two simultaneous cycles of 10 and 12 years (60 is the lowest common multiple of 10 and 12). The years in both of these Chinese cycles carried names. The cultures of Central America and of India used several cycles of years; some are exceedingly long.

15.1.7 Historical Eras and Chronology

Astronomical cycles with a period greater than a year have rarely been used in calendars. Instead, years have generally been numbered from some important event, secular or religious. Thus, regnal dates count the years from the accession of a ruler and the numbering starts afresh with each new ruler. Other calendars count years from an important event, maybe in the remote past, which initiates an era. Thus, the Islamic calendar counts the years from the date of the flight of the Prophet from Mecca to Medina and the Christians count from the presumed birth of Christ. In other calendars, Chinese, Indian, Mayan, years were sometimes counted in cycles of various lengths; the cycles themselves may also be counted.

The first day of the first month of the year numbered 1 is sometimes called the epoch of the calendar. Table 15.2 shows the starting year of a few of the eras of historical importance.

15.1.8 The Christian Era

The epoch of the Christian calendar is usually, but wrongly, taken to be the birth of Christ. This epoch was established by the sixth-century scholar Dionysius Exiguus who was compiling a table of dates of Easter. An existing table covered the nineteen-year period denoted 228–247, where years were counted from the beginning of the reign of the Roman emperor Diocletian. Dionysius continued the table for a nineteen-year period, which he designated Anno Domini Nostri Jesu Christi 532–550. Thus, Dionysius’ Anno Domini 532 is equivalent to Anno Diocletiani 248. In this way a correspondence was established between the new Christian

Table 15.2 Some eras

Name of era	Year of epoch	Used by	Event commemorated
Byzantine Era ^a	−5508	Byzantium	Creation of the world
Anno Mundi (A.M.) ^a	−3760	Judaism	Creation of the world
Kali-yuga	−3101	India	Mean conjunction of planets
^b	−2636	Chinese	Traditional start of 60 year cycles
Ab Urbe Condita (A.U.C.) ^a	−752	Roman Empire	Legendary foundation of Rome
The Era of Nabonassar ^a	−746	Alexandria, Early astronomers	Accession of Babylonian King Nabonassar
Jimmu Tenno ^a	−659	Japan	Accession of Emperor Jimmu Tenno; adoption of Chinese calendar
Seleucid Era ^a	−312	Selucid Empire	Foundation of Empire by Seleucis
Anno Domini (A.D. or C.E)	1	Christianity	Approximate date of Birth of Christ
Saka Era ^a	78	India	Uncertain; possible accession of King Salivahana
Era of Diocletian ^a	283	Coptic & Ethiopian churches; Roman Empire	Accession of Roman Emperor Diocletian
Era of the Hegira (A.H.) ^a	622	Islam	Prophet's flight from Mecca
Republican Era (E.R.)	1792	France	Foundation of the Republic
Badi Era (B.E.)	1844	Bahá'í	Declaration of the Bab

^a The dates of the start of the year in these eras are currently given in *The Astronomical Almanac*.

^b The Chinese used no era as such but counted years in cycles of 60. The cycles were sometimes counted with the first starting in −2636.

era and an existing system associated with historical records. However, Dionysius did not use an accurate date for the birth of Christ and scholars generally believe that Christ was born some years before A.D. 1, but the historical evidence is not adequate to allow a definitive dating.

Years after the epoch of the Christian calendar are traditionally labeled: A.D. (Anno Domini) but the label C.E. (Common Era) is preferred by some. Likewise the traditional label of years before A.D. 1 has been B.C. (Before Christ) but some prefer B.C.E. (Before Common Era).

It is often possible to assign a date in a calendar to days preceding the epoch or the date on which it was first used. Such dates are termed *proleptic* dates. Bede, the eighth-century English historian, began the practice of counting years backward from A.D. 1 (see

Colgrave and Mynors 1969). In this system, the year A.D. 1 is preceded by the year 1 B.C., with no intervening year 0. Because of this numerical discontinuity, this “historical” system is cumbersome for comparing ancient and modern dates. I have used +1 (etc.) to designate A.D. 1 (etc.). The year +1 is naturally preceded by year 0, which is preceded by year −1. Since the use of negative numbers developed slowly in Europe, this system of dating was delayed until the eighteenth century, when it was introduced by the astronomer Jacques Cassini (Cassini 1740).

15.1.9 Dates

The date of any day is specified by giving the year and the month in which it falls and its day of the month and maybe the day of the week. There are many conventions concerning the order in which these are presented. We shall specify here dates by giving the day of the week, the year number, the name of the month and the day of the month in that order.

Note that in nearly all calendars, the first day of a month is day 1—rather than day 0; likewise, the first month of a year is month 1—rather than month 0. This is in contradistinction to other systems for specifying time; thus the first hour of a day is hour 0—rather than hour 1.

We shall use, following the practice of historians, the Julian calendar (see § 15.3.4) for dates before 1582 and the Gregorian for dates after, but it is usual to specify the calendar used for dates between 1582 and 1800. We shall not always label the years with either A.D./B.C. or C.E./B.C.E. Instead a positive year number should be read to imply a year in the Common Era (A.D.), whilst a negative year implies a year before that (B.C.). Thus, a year 2000 means A.D. 2000 whilst −1000 implies 1001 B.C.. In general negative year, −Y is equivalent to 1 − Y B.C.. We will, however, use the terms A.D. or B.C. when referring to centuries.

The time of day of the start of a day varies in different Calendars. Thus, the Gregorian day runs from midnight to midnight, but the Islamic and Jewish days start some six hours earlier and run from dusk (at a nominal 6 p.m.) to dusk. This raises the point as to which, for example, Gregorian day or date corresponds to a given Islamic or Jewish date. The usual convention, which we follow here and in *The Astronomical Almanac*, is that dates in different calendars correspond when they contain the same noon. Thus the Islamic and Jewish days start some six hours before the corresponding Gregorian day. In some Indian calendars the day starts at dawn, some six hours later.

15.1.10 Julian Day Numbers and Julian Dates

Dating systems in medieval Europe caused considerable confusion. Some states used regnal dating (with a plethora of local rulers); others used the Anno Domini system. Moreover different states used different days to start their years. Although the Romans had used January 1 to start their years, Christendom often used March 25th, though other days were sometimes used.

In the sixteenth century, Joseph Justus Scaliger (1540–1609) tried to resolve the patchwork of historical eras by dating every event according to a single system (Scaliger 1583). Instead of introducing negative year counts, he sought an initial epoch before any historical record. His approach utilised three calendrical cycles: the 28-year solar cycle (the period after which weekdays and calendar dates repeat in the Julian calendar), the nineteen-year cycle of

golden numbers (the period after which the Moon's phases approximately repeat on the same calendar dates), and the fifteen-year indiction cycle (the Roman tax cycle).

Scaliger could, therefore, characterise a year by a triplet of numbers (**S**, **G**, **I**); **S**, the number of the year in the solar cycle, runs from 1 to 28; **G**, the golden number of the year, runs from 1 to 19; **I**, the number of the year in the Indiction cycle runs from 1 to 15. He noted that a given combination would recur after 7980 ($= 28 \times 19 \times 15$) years. This he called a *Julian Period*, because it was based on the Julian calendar year. For his initial epoch, Scaliger chose the year in which **S**, **G** and **I** were all equal to 1. He knew that the year 1 (A.D. 1) had **S** = 10, **G** = 2 and **I** = 4 and worked out that the combination (1,1,1) occurred in the year -4712 (4713 B.C.) which was year 1 of Scaliger's Julian period.

Although Scaliger's original idea was to introduce a count of years, nineteenth-century astronomers adapted this system to create a count of days elapsed since the beginning of the Julian period. John Herschel (1849) gave a thorough explanation of the system and provided a table of "Intervals in Days between the Commencement of the Julian Period, and that of some other remarkable chronological and astronomical Eras."

Astronomers have extended this idea by appending to the day number, the time of day expressed as a decimal fraction of a day. The resulting numbers called Julian dates define any instant of time. Thus 2 451 545.0 corresponds to noon on 2000 January 1 as observed at a meridian of 0° (Greenwich) in the Gregorian calendar. At a meridian of 0°, the day 2000 January 1 began 12 hours earlier at 2 451 544.5. Further east, the day begins earlier and further west, later.

It is important to note these Julian dates specify an instant in *Universal Time* (UT), not local time. A day in the Gregorian calendar begins at midnight, the world over, at an instant that varies with longitude, and hence is specified by different Julian dates. This is inconvenient when considering calendars. It is, therefore, useful to define a Julian Day Number, **J**, (to be distinguished from a Julian date) which is a whole number, and which may be used to label each of the days. The Julian Day Number of Saturday 2000 January 1 is then 2 451 545 and of Monday -4712 January 1 it is 0. URL[7] provides a useful program for interconverting dates and Julian dates.

15.1.11 Luni-solar Calendars

It may be noted that 19 astronomical years and 235 lunations both contain about 6939 days. This was realised in Babylon in about the 5th century B.C., and gave rise to a calendar usually ascribed to Meton of Athens (5th century B.C.). In this, each year of a 19-year cycle contains 12 months of alternately 29 and 30 days, but 7 of the 19 years contain a 13th intercalated month of 30 days. Thus the 19 years contain 235 months or 6936 days. The average number of days per year is about 365.05 ($= 6936/19$) and the average number of days per month is about 29.51. In the short term, such a calendar keeps, on average, in rough synchrony with both the Sun and the Moon. The accuracy of this Metonic calendar can be improved by judicious intercalations of extra days, and a variant of it is used in both the Jewish and the Christian Ecclesiastical calendars.

The luni-solar calendars of China and India similarly interpolate leap months, but use astronomical considerations to decide when to intercalate and when to begin a year or a month. The months themselves may begin on the day of a conjunction of the Moon (new

moon) and are thus synchronised with the lunations, but these calendars also make use of solar months. These are defined by dividing the ecliptic into twelve sections, analogous to the sign of the Zodiac, and separated by 30° of solar longitude. The temporal lengths of both the lunar months and of the solar months vary throughout the year. Thus, although the starting dates of the solar and lunar months generally alternate, so that each lunar month contains the start of just one solar month, there are years in which a lunar month contains the start of two solar months (i.e., the solar month is entirely contained in a lunar month) or of no solar month (i.e., the lunar month is entirely contained in a solar month). The points at which extra months are intercalated are determined by such months. This results in the intercalation of about 7 extra months in 19 years and achieves a result similar to that of the Metonic calendar.

15.1.12 Accuracy of Calendars

The question of accuracy for observational calendars does not arise. The accuracy of an astronomical calendar depends on the adequacy of the underlying astronomical theory and the precision of the celestial observations on which it is based. Although arithmetic calendars do not explicitly refer to astronomical cycles, they are usually founded on an attempt to maintain their years in synchrony with the tropical year or their months in synchrony with the lunar cycle. These attempts can never be wholly accurate if only because the astronomical periods change with time, albeit slowly. To assess their accuracy, the synodic period may be compared to the average length of the calendar month and the mean tropical year (or the vernal equinox or winter solstices year) may be compared to the average length of the calendar year. A useful parameter is the number of years required for a calendar cycle to become one day out of phase with the corresponding astronomical cycle in some particular year. Bear in mind that the lengths of these astronomical cycles are themselves slowly changing (see Table 15.1).

High precision in the average lengths of the months was achieved quite early on account of the ease with which the mean synodic period can be measured. The precise measurement of the length of the year (solstice to solstice or equinox to equinox) seems to have presented a greater challenge. The most accurate solar calendar in general use is the Gregorian calendar in which there are 146 097 days in 400 years or 365.2425 days/year. Today, in A.D. 2000, the Gregorian calendar year is slightly longer than the mean vernal equinox year (365.24237 days) and, neglecting its change with time, would be one day out relative to the vernal equinox year after about another 8000 years. Various proposals have been made (Delambre 1821; Herschel 1849) to improve this accuracy but none take count of the slowly changing length of the year and day.

At the other extreme, the ancient Egyptian civil calendar contained 365 days. This lagged a further day behind the tropical year every four years or by 365 days in about 1500 years.

15.2 The Ancient Egyptian Calendar

15.2.1 History of the Egyptian Calendars

Ancient Egyptian civilisation, from the start of the Old Kingdom to the annexation of Egypt by Augustus, lasted some 2500 years. It employed several calendars for religious and for civil purposes, but it is not known when these were first instituted. The year of the civil calendar contained a constant 365 days divided into 12 months of 30 days each, followed by

Table 15.3 Months in the ancient Egyptian calendar

Month	Month
1. Thoth	7. Phanemoth
2. Paophi	8. Pharmouti
3. Athyr	9. Pachons
4. Cohiac	10. Payni
5. Tybi	11. Epiphi
6. Mesir	12. Messori

The five epagomenal days followed Messori.

5 *epagomenal* days belonging to no month. The names of the months are shown in Table 15.3. The year began on Thoth 1. Since 365 is a poor approximation to the number of days in the year, the start of the year cycled slowly through the seasons with a period of about 1500 years. There was no Egyptian Era, instead the years were counted from the accession of each Pharaoh in regnal fashion, but Ptolemy used the era of Nabonassar (E.N.) (see § 15.1.7).

The Roman writer Censorinus noted that the Julian calendar date 139 July 20 (Julian Day Number 1 772 028) was Thoth 1 in the Egyptian calendar. This day started year 887 E.N.

The Egyptian calendar was reformed by the Roman Emperor Augustus in 23 B.C.. He decreed that a sixth epagomenal day should be inserted once every four years to keep it in synchrony with the Julian calendar. This modified calendar is sometimes called the Alexandrian calendar.

This Alexandrian calendar is not extinct even today. Calendars closely similar to it are used by the Coptic Church in modern Egypt and in Ethiopia. It was also used by a variety of cultures and religions including The Zoroastrian Parsees, the Armenian Church and the French Republicans after their revolution.

15.2.2 Rules of the Egyptian Calendar

The Egyptian calendar has 12 month of 30 days in each; the five epagomenal days may be considered to form a thirteenth month of five days. Thus, each year has exactly 365 days. The day Thoth 1, 1 E.N. was Wednesday, February 26, –746 in the proleptic Julian Calendar and had a Julian Day Number 1 448 638.

The Alexandrian version has an extra leap day, a sixth epagomenal day, intercalated at the end of the thirteenth month once every four years.

15.3 The Roman and Julian Calendars

15.3.1 Introduction

The Roman calendar was probably originally lunar and relied on intercalations, which were applied haphazardly. It underwent several alterations in which the names and lengths of the months were changed, although some of the details are lost. By –45, there were 12 months

mostly with names similar to those in the current Gregorian calendar, though the numbers of days in several were different. The year started on Januarius 1.

At one time the years were indicated by a regnal system that mentioned the names of the current Consuls. Later, in the first century B.C., the years were numbered from the presumed date of the foundation of Rome, -752 ; these are termed *ab urbe condita* (A.U.C). Later still the Emperor Diocletian instituted an era which started with his accession in 283.

15.3.2 Divisions of the Roman Months

The Roman months were divided into three unequal sections, which may, when the Roman calendar was lunar, have marked the phases of the Moon. The last days of each section, which we may call the division points, were called the Kalends (*Kalendae*), the Nones (*Nonae*) and the Ides (*Idus*). The Kalends was the first day of the month; the Ides, the thirteenth of the month, except in March, May, July, and October, when it was the fifteenth day. The Nones was always eight days before the Ides (see Table 15.4). The days preceding the division points were termed Prid. Kal. (*pridie* = day before), Prid. Non. and Prid. Id. respectively. The remaining days were numbered backwards from the division days as indicated in Table 15.4 and labelled with the month in which the division point fell. For example, January 4 was termed Prid. Non. Jan. and January 11 was III Id. Jan. Confusingly, but logically, the days preceding the Kalends which fell in the previous month were labelled by the month in which the division point fell, rather than the month to which they belonged; thus December 30 was called III Kal. Jan. This Roman system is occasionally used, even to this day.

15.3.3 Caesar's Reform

By -46 , the Roman calendar had gone badly awry; the months no longer followed the lunations and the year had lost step with the cycle of seasons (see Michels 1978; Bickerman 1980). This state of affairs was reformed by Julius Caesar (107–44 B.C.), who took the advice of the Alexandrian astronomer, Sosigenes.

Caesar first inserted 90 days into the year -45 to bring the months of the Roman calendar back to their traditional places with respect to the seasons. This year has been called “the last year of confusion.” He next changed the length of some of the old months. Finally he made provision for an intercalary day to be inserted every four years. This intercalary day (see § 15.3.5) was inserted before VI Kal. Mar. and termed Bis VI Kal. Mar.; it fell between VII Kal. Mar. (February 23) and VI Kal. Mar. (February 25th). Years in which this was done were termed *bissexile years*.

This new Roman calendar, the Julian calendar, was thus a solar calendar. The average length of the calendar year was 365.25 days so that the vernal equinox fell back from March 21 by one day in about 130 years. Nevertheless it served as a standard for European civilisation until the Gregorian Reform of 1582.

Following Caesar's death, the Roman calendrical authorities misapplied the leap-year rule, with the result that the intercalary day was inserted every third, rather than every fourth, year. This error was rectified by Augustus in -8 . The details of this correction are uncertain but it is likely that he decreed that there should be no intercalation in the years -7 to $+6$ and that there should be intercalary days (leap days) inserted every fourth year from then on (i.e., in years 8, 12, 16 . . .).

Table 15.4 Roman dating in the Julian calendar

	Januarius Augustus December	Februarius	Martius Maius Julius October	Aprilis Junius September November
1	Kalendae	Kalendae	Kalendae	Kalendae
2	IV Non.	IV Non.	VI Non.	IV Non.
3	III Non.	III Non.	V Non.	III Non.
4	Prid. Non.	Prid. Non.	IV Id.	Prid. Non.
5	Nonae	Nonae	III Non.	Nonae
6	VIII Id.	VIII Id.	Prid. Non.	VIII Id.
7	VII Id.	VII Id.	Nonae	VII Id.
8	VI Id.	VI Id.	VIII Id.	VI Id.
9	V Id.	V Id.	VII Id.	V Id.
10	IV Id.	IV Id.	VI Id.	IV Id.
11	III Id.	III Id.	V Id.	III Id.
12	Prid. Id.	Prid. Id.	IV Id.	Prid. Id.
13	Idus	Idus	III Id.	Idus
14	XIX Kal.	XVI Kal.	Prid. Ides	XVIII Kal.
15	XVIII Kal.	XV Kal.	Ides	XVII Kal.
16	XVII Kal.	XIV Kal.	XVII Kal.	XVI Kal.
17	XVI Kal.	XIII Kal.	XVI Kal.	XV Kal.
18	XV Kal.	XII Kal.	XV Kal.	XIV Kal.
19	XIV Kal.	XI Kal.	XIV Kal.	XIII Kal.
20	XIII Kal.	X Kal.	XIII Kal.	XII Kal.
21	XII Kal.	IX Kal.	XII Kal.	XI Kal.
22	XI Kal.	VIII Kal.	XI Kal.	X Kal.
23	X Kal.	VII Kal.	X Kal.	IX Kal.
24	IX Kal.	VI Kal.	IX Kal.	VIII Kal.
25	VIII Kal.	V Kal.	VIII Kal.	VII Kal.
26	VII Kal.	IV Kal.	VII Kal.	VI Kal.
27	VI Kal.	III Kal.	VI Kal.	V Kal.
28	V Kal.	Prid. Kal.	V Kal.	IV Kal.
29	IV Kal.		IV Kal.	III Kal.
30	III Kal.		III Kal.	Prid. Kal.
31	Prid. Kal.		Prid. Kal.	

15.3.4 The Julian Calendar in Medieval Europe

Through the Middle Ages the use of the Julian calendar evolved and acquired local peculiarities that continue to snare the unwary historian. There were variations in the initial epoch for counting years, the date for beginning the year, and the method of specifying the day of the month. Not only did these vary with time and place, but also with purpose. Different conventions were sometimes used for dating ecclesiastical records, fiscal transactions, and personal correspondence.

Caesar designated January 1 as the beginning of the year. However, other conventions flourished at different times and places. The most popular alternatives were March 1, March 25, and December 25. This continues to cause problems for historians, since, for example, 998 February 28 as recorded in a city that began its year on March 1, would be the same day as 999 February 28 of a city that began the year on January 1.

By the eleventh century, consecutive counting of days from the beginning of the month came into use, displacing the Roman system of divisions. Local variations continued, however, including counts of days from dates that commemorate local saints. A uniform standard for recording dates was instituted only after the inauguration and spread of the Gregorian calendar. The leap day became February 29 instead of the day after February 23.

Cappelli (1930), Grotefend and Grotefend (1941), and Cheney (1981) offer guidance through the maze of medieval dating.

Today the Julian calendar continues to be used by chronologists. The Julian proleptic calendar is formed by applying the rules of the Julian calendar to times before Caesar's reform. This provides a simple chronological system for correlating other calendars and serves as the basis for the Julian Day Numbers (see § 15.1.10).

15.3.5 Rules for the Julian Calendar

The month names and the number of days in each are listed in Table 15.5 for the Julian calendar. The months in the Gregorian calendar are identical.

Table 15.5 Months of the Julian and Gregorian calendars

Month			Month ^a		
1.	January (Januarius)	31	7.	July (Julius ^b)	31
2.	February (Februarius)	28 ^c	8.	August (Augustus ^d)	31
3.	March (Martius)	31	9.	September (September)	30
4.	April (Aprilis)	30	10.	October (October)	31
5.	May (Maius)	31	11.	November (November)	30
6.	June (Junius)	30	12.	December (December)	31

^a These months were at one time the 5th, 6th, 7th, 8th, 9th and 10th aptly named months of the Roman year.

^b Originally called Quintilis but renamed in honour of Julius Caesar.

^c In a leap year, February has 29 days

^d Originally called Sextilis but renamed in honour of the emperor Augustus.

A Julian calendar year thus contains 365 days or 366 in a leap year. Years are, today, counted from the initial epoch defined by Dionysius Exiguus (see § 15.1.8). A leap day, February 29, is intercalated after February 28 in leap years. Leap years are divisible by 4, e.g., +2000 was a leap year but +2001 was not. For this purpose, year 0 (or 1 B.C., see § 15.1.9) is considered to be divisible by 4.

The epoch of the calendar (Saturday, 1 January 1) had a Julian Day Number 1 721 424

15.4 The Gregorian Calendar

15.4.1 History of the Gregorian Reform

The early Christians church adopted the Julian calendar and used it as the basis for calculating the date of Easter following the recommendation of the Council of Nicea held in 325. March 21 was assumed to be the date of the equinox and the Metonic cycle was used as the basis for calculating the lunar phases.

By the thirteenth century it was realised that the true vernal equinox had regressed from March 21 to an earlier day in March. Over the next four centuries, scholars debated the “correct” time for celebrating Easter and the need for a reform of the calendar. The Church made intermittent attempts to remedy the matter, without taking action.

By the sixteenth century the equinox had shifted to March 11, and astronomical new moons were occurring four days before the dates assumed in the calculation of the date of Easter. At the behest of the Council of Trent, Pope Pius V introduced a new Breviary in 1568 and Missal in 1570, both of which included adjustments to the lunar tables and the leap-year system. Pope Gregory XIII, who succeeded Pope Pius in 1572, convened a commission to consider reform of the calendar, since he considered his predecessor’s measures inadequate.

This commission adopted a proposal suggested by Aloysius Lilius (1510–1576) and issued a report written by Christopher Clavius (1537–1612) which resulted in a papal bull, “*inter Gravissimas*” signed by Gregory XIII on 1582 February 24.

Ten days were deleted from the calendar, so that 1582 October 4 was followed by 1582 October 15, thereby causing the vernal equinox of 1583 and subsequent years to return to about March 21. The rules for intercalating a leap day were changed, and a new method of calculating the dates of Easter was introduced. The new method of intercalation meant that a period of 400 years contained 146 097 days. Since 146 097 is divisible by 7, the Gregorian civil calendar exactly repeats after 400 years. The average number of days in a year is 365.2425. (see Table 15.1). It would be one day out of synchrony with the vernal equinox year after about another 3000 years (if the year were to remain unchanged in length).

This Gregorian calendar today serves as an international standard for civil use. In addition, it regulates the ecclesiastical calendar of the Catholic and Protestant churches but was rejected by the Orthodox churches. It was first promulgated and adopted throughout the Roman Catholic world; the Protestant states initially rejected it, but one by one they accepted it over the coming centuries. England (and her then American colonies) finally accepted the change in 1752 when 11 days were lost; contrary to popular belief there is no evidence for riots in protest at this (Poole 1998). The Eastern Orthodox churches continue to use the Julian calendar with the traditional lunar tables for calculating Easter. Because the purpose of the Gregorian calendar was to regulate the cycle of Christian holidays, its acceptance in the non-Christian world was initially not at issue.

Detailed information about the Gregorian reform may be found in the collection of papers resulting from a conference sponsored by the Vatican to commemorate its four-hundredth anniversary (Coyne, Hoskin and Pedersen 1983).

15.4.2 Rules for the Civil Use of the Gregorian Calendar

The Gregorian calendar uses the same months with the numbers of days as its predecessor, the Julian calendar (see Table 15.5). Days are counted from the first day of each month.

Years are counted from the initial epoch defined by Dionysius Exiguus (see § 15.1.8), and each begins on January 1. A common year has 365 days but a leap year has 366, with an intercalary day, designated February 29, preceding March 1. Leap years are determined according to the following rule:

Every year that is exactly divisible by 4 is a leap year, except for years that are exactly divisible by 100, but these centurial years are leap years if they are exactly divisible by 400.

As a result, the year 2000 was a leap year, whereas 1900 and 2100 are not.

The epoch of the Gregorian calendar, (1 January 1) was Monday, 1 January 3 in the Julian calendar or Julian Day Number 1 721 426.

15.4.3 Rules for the Ecclesiastical Calendar

The ecclesiastical calendars of Christian churches are based on cycles of movable and immovable feasts. Christmas is the principal immovable feast, with its date set at December 25. Easter is the principal movable feast, and dates of most other movable feasts are determined with respect to Easter. However, the movable feasts of the Advent and Epiphany seasons are Sundays reckoned from Christmas and the Feast of the Epiphany, respectively.

In both the Julian and Gregorian calendars, the date of Easter is defined to occur on the Sunday following the ecclesiastical full moon that falls on or next after March 21. This should not be confused with the popular notion that Easter is the first Sunday after the first full moon following the vernal equinox. In the first place, the vernal equinox does not necessarily occur on March 21. Secondly, the ecclesiastical full moon is not the astronomical full moon. It is based on tables that do not take into account the full complexity of lunar motion. As a result, the date of an ecclesiastical full moon may differ from that of the true full moon. However, the Gregorian system of leap years and lunar tables prevents any progressive departure of the tabulated date from the astronomical phenomena.

The ecclesiastical full moon is defined as day 14 of a lunation, where the day of the ecclesiastical new moon is counted as day 1. The tables are based on the Metonic cycle, in which 235 mean synodic months contain 6939.688 days. Since nineteen Gregorian years contain 6939.6075 days, the dates of Moon's phases in a given year will recur on nearly the same dates nineteen years later. To prevent the 0.08-day difference between the cycles from accumulating, the tables incorporate adjustments to synchronise the system over longer periods of time. Additional complications arise because the ecclesiastical lunations are of 29 or 30 integral days. The entire calendar involves a cycle 5 700 000 years containing 2 081 882 250 days, which are equated to 70 499 175 lunations. After this period, the dates of Easter repeat themselves.

The date of Easter as calculated in the Gregorian calendar is used by the Catholic and most Protestant churches. However, the Eastern Orthodox churches still calculate the date of Easter in the Julian calendar, though some such churches have, in this century, modified the original method of calculation.

15.4.4 Calculation of the Date of Easter Sunday

The date of Easter was calculated using three parameters.

The first of these is the Dominical (or Sunday) Letter. This defines the positions in a year of the Sundays. It is obtained by labelling the days of the year consecutively with the seven letters A to G. And for the next 7, again A–G and so on. The Dominical Letter is the letter so assigned to the first Sunday (and every other). It changes from year to year.

Equating A with 1, B with 2 etc., the Dominical Letter for year Y is given by the equivalent to the number, N , in:

$$N = 7 - \text{mod}(Y + Y/4 + 4, 7) \quad \text{in the Julian calendar} \quad (15.3)$$

or

$$N = 7 - \text{mod}(Y + Y/4 - Y/100 + Y/400 - 1, 7) \quad \text{in the Gregorian calendar.} \quad (15.4)$$

The second, the Golden number, is the position (1 to 19) of a year in the 19 year Metonic cycle. If Y is the year in either the Julian or Gregorian calendar, the golden number, G , is given by:

$$G = 1 + \text{mod}(Y, 19). \quad (15.5)$$

The third is the Epact of a year. It is the age in days (0 to 29) of the ecclesiastical moon on the first day of the year (January 1).

For the Dionysian canon (tied to the Julian calendar), the epact, E , of a year with a Golden number G is given by:

$$E = \text{mod}(11 * G - 3, 30). \quad (15.6)$$

In the Gregorian canon, the calculation of the epact is more complicated. First the basic formula for the epact is changed to:

$$E = \text{mod}(11 * G - 10, 30). \quad (15.7)$$

But this must be modified by the addition of the so called solar (SOL) and lunar (LUN) equations.

The solar equation firstly makes an adjustment needed to return the date of the vernal equinox to its traditional date close to March 21. Secondly it makes an adjustment of the decreased length of the Gregorian year. It is given by:

$$\text{SOL} = H - H/4 - 12. \quad (15.8)$$

The lunar equation likewise gives an adjustment to correct the average length of a lunation to value more close to that of the real Moon. It is given by:

$$\text{LUN} = (H - 15 - (H - 17)/25)/3. \quad (15.9)$$

In both these expressions, $H = Y/100$ where Y is the year.

A further adjustment of 1 day is required of the Golden number if $E = 25$ and $G \geq 12$ or $E = 24$. This is to maintain a traditional requirement that no two new moons fell on the same day of the year in a Metonic cycle of 19 years. It may be calculated as V in

$$V = E/24 - E/25 + (G/12) * (E/25 - E/26) \quad (15.10)$$

so that the adjusted Epact is

$$E = \text{mod}(11 * G - 10, 30) - \text{mod}(\text{SOL} - \text{LUN}, 30) + V. \quad (15.11)$$

The day of Easter Sunday is then given as a Day of March, S , (i.e., counting from March 1 as day 1) in terms of this modified epact E by:

$$S = R + \text{mod}(7 + N - C, 7) \quad (15.12)$$

where

$$C = 1 + \text{mod}(R + 2, 7) \quad (15.13)$$

and

$$R = 45 - E \quad \text{if } E < 24 \quad (15.14)$$

$$\text{or } R = 75 - E \quad \text{if } E \geq 24 \quad (15.15)$$

See § 15.11.1 for a full explanation of the mathematical notation used here. Details of the calculation are discussed at length by Butcher (1877), Oudin (1940) and Richards (1998).

15.5 The Jewish Calendar

15.5.1 History of the Jewish Calendar

The codified Jewish calendar as we know it today is generally considered to date from about 359, though the exact date is uncertain.

Jewish calendrical practices before that are uncertain. The earliest evidence indicates a calendar based on observations of the phases of the new moon at Jerusalem. Since the Bible mentions seasonal festivals, there were probably intercalations. There was probably an evolution of conflicting calendrical practices.

The Babylonian exile, in the first half of the sixth century B.C., greatly influenced the Jewish calendar. The names of the months are very similar to those in the Babylonian calendar and the practice of intercalating months may have been learnt from the Babylonians.

During the period of the Sanhedrin the calendar was observational; months began with sightings of the crescent of the new moon. A committee of the Sanhedrin met to evaluate reports of sightings. If sightings were not possible, the new month was begun 30 days after the beginning of the previous month. Decisions on intercalation were influenced, if not determined entirely, by the state of vegetation and animal life. Although eight-year, nineteen-year, and longer-period intercalation cycles may have been instituted at various times prior to Hillel II, there is little evidence that they were employed consistently over long periods. The Sanhedrin was entrusted to run the calendar and only its members knew how to do it.

After the Diaspora, this arrangement became unworkable; by the time news of sightings at Jerusalem reach outlying Jewish communities, the start of the months was well past. The patriarch Hillel II is credited with reforming this state of affairs by disseminating codified rules, which anybody, anywhere, could follow.

The exact details of Hillel's calendar have not survived, but it is generally considered to include rules for intercalation over nineteen-year cycles. Up to the tenth century A.D., however, there was disagreement about the proper years for intercalation and the initial epoch for reckoning years.

The modern Jewish calendar is the official calendar of Israel and is the liturgical calendar of the Jewish faith.

15.5.2 Rules for the Modern Jewish Calendar

The day of the Jewish calendar is divided into 24 hours and each hour into 1080 *halakim* (*hk*). Thus, there are 10 seconds in 3 halakim. The day begins at 18.00 P.M., that is six hours in advance of our current calendrical day and this is counted as 0 hr 0 hk in Jewish timekeeping. The days of the week are numbered from Sunday (1) to Saturday, the Sabbath (7).

As it exists today, the Jewish calendar is a luni-solar calendar that is based on calculation rather than observation and is based on the Metonic calendar. Its regulation depends on three levels of abstraction. First, there is the real astronomical moon; secondly, a highly regular but fictitious moon whose behaviour is close to the average behaviour of the real Moon and which defines the astronomical calendar. Each month and year of the latter starts at the instant of conjunction of the fictitious moon; this is termed a *Molad*. Thirdly, there is the civil calendar; the months and years of this start on, or up to two days after, the *molad*. Thus, the start of a civil year may be postponed from the date of its *molad* by the application of four rules (*dehiyyot*).

The epoch of the calendar is taken to occur in the same year, -3760 , as the Creation as described in Genesis. This year was calculated in about the 10th century by Maimonides from data in the Bible, which gives the generation times of the Patriarchs from Adam, and important historical dates culminating in the sack of Jerusalem by the Romans which can be dated historically. The results of such calculations are controversial but Maimonides' value is accepted by the Jewish religion.

Years counted from this epoch define the Mundane Era (Anno Mundi, A.M.) and it may be noted that the Jewish year, *Y*, currently begins in the autumn of the Gregorian or Julian year: $Y - 3761$. The epoch of the calendar, the first day of Tishri is 1 A.M. is on Monday, -3760 October 7 in the proleptic Julian calendar with a Julian Day Number of 347 998. This epoch is not the date of the creation itself; this is taken to have occurred in the next September.

The years are counted in cycles of 19 years, and in this period there are 12 common years of 12 months apiece and 7 leap years each containing 13 months; thus there are 235 months in the 19-year cycle. The leap years are now fixed as the 3rd, 6th, 8th, 11th, 14th, 17th and 19th year of each cycle

The total period of this 19-year cycle is 6939 days, 16 hours, 595 halakim (=6939.689 590 days) so that the average number of days in a year is 365.24682. Thus, the average start of the Jewish year is currently falling behind the mean tropical year by about 1 day in 200 years.

Each civil year is deemed to start with the first day of the first month, Tishri. The first (fictitious) *molad*, which initiates year 1, and which is the first year of a 19 year cycle, is

Table 15.6 Number of days in the six categories of the Jewish year

	Common year	Leap year
Deficient	353	383
Regular	354	384
Abundant	355	385

Table 15.7 Months in the Jewish calendar

	Month		Month		
1.	Tishri	30	7.	Nisan	30
2.	Heshvan	29 ^a	8.	Iyar	29
3.	Kislev	30 ^b	9.	Sivan	30
4.	Tebet	29	10.	Tammuz	29
5.	Shebat	30	11.	Ab	30
6.	Addar	29 ^c	12.	Elul	29

^a In an abundant year, Heshvan has 30 days.

^b In a deficient year, Kislev has 29 days.

^c In a leap year, Addar has 30 days; it is followed by Addar II with 29 days.

deemed to have taken place on the Monday, October 7, –3760 at 11:20 P.M. or Tishri 1, A.M. 1 at 5 hr, 204 hk; it is a simple matter to calculate the *molad* of Tishri of any succeeding year by adding an appropriate multiple of the interval between successive *moladot* to the time of this first *molad*.

Successive *moladot* occur at fixed intervals of 29 days 12 hr and 793 hk (= 29.530 594 days). They do not coincide with the phases of the real astronomical Moon, but they should remain in average synchrony for a very long time. Calendrical dates are worked out, independent of longitude using local times.

The rules of postponement and the insertion of leap month imply that the number of days in a year varies. There are six types of calendar year, according to the number of days in them. Firstly, a year may be common or leap; common years contain 12 months and leap years contain 13. Furthermore any year may be deficient, regular or abundant. The numbers of days in the six types of year are shown in Table 15.6. The names of the months and the number of days therein are shown in Table 15.7 Generally the number of days in the month alternate between 30 and 29.

15.5.3 Rules for Postponement (Dehiyyot)

Below are the four rules that determine whether Tishri 1 should be postponed until after its *molad*. The first three are sometimes termed astronomical rules while the fourth is termed a political rule. We also give below the reason underlying each rule. Only one of the astronomical postponements should be applied. Rule (4) is applied last but its application may be required after the application of one of the other three to give a second postponement.

1. If the Molad of Tishri falls at or after 18 hr 0 hk (i.e., noon), Tishri 1 is postponed by one day. If this causes Tishri 1 to fall on day 1, 4, or 6, (Sunday, Wednesday or Friday) then Tishri 1 is postponed an additional day to satisfy Rule (4).

Before the introduction of the new calendar, the Molad of Tishri was determined by the first sighting of the new moon; this would always be close to the Sun. If the *molad* occurred before dusk (nominally 6:00 P.M.), it might not be visible and would be taken to fall on the next day. Rule (1) preserves this traditional feature.

2. If the Molad of Tishri of a common year (i.e., of twelve months) falls on a Tuesday (day 3) at or after 9 hr, 204 hk, then Tishri 1 is postponed one day to the Wednesday (day 4). Rule (4) would then require a further postponement to the Thursday.

Should this rule apply to year Y, it may be calculated that the *Molad* of the succeeding year, Y+1, would fall twelve months later on a Saturday on or after 18 hr, 0 hk. Year Y+1 would then be postponed one day by Rule (1) and a further day by Rule (4). This would then mean that year Y would have an unacceptable 356 days. This is avoided by postponing the start of year Y by one day so that it would have 355 days.

3. If the Molad of Tishri of a common year following a leap year falls on a Monday (day 2) at or after 15 hr, 589 hk, then Tishri 1 is postponed one day to the Tuesday (day 3).

Should this rule apply to the common year Y, it may be calculated that the *Molad* of the previous leap year, Y−1, would occur (thirteen months earlier) on a Tuesday on or after 18 hr, 0 hk. Year Y−1 would thus have been postponed by one day by Rule (1) and then a further day by Rule (4). This would then mean that year Y−1 had an unacceptable 382 days. This is avoided by postponing the start of year Y by one day so that year Y−1 would have 383 days.

4. If the Molad of Tishri falls on a Sunday, Wednesday or Friday (days 1, 4 or 6), then Tishri 1 is postponed by one day.

This (political) rule prevents Hoshanna Rabba (Tishri 21) from occurring on the Sabbath and Yom Kippur (Tishri 10) from occurring on the day before or after the Sabbath. Work (including the preparation of food and the burial of the dead,) is forbidden on Yom Kippur and the Sabbath; this rule prevents there being two consecutive days on which work is forbidden. Work is necessary on Hoshanna Rabba and Rule (4) eliminates any conflict.

A thorough discussion of both the functional and religious aspects of the *dehiyyot* is provided by Cohen (1981).

15.5.4 Determining Tishri 1

The calendar year begins with the first day of Rosh Hashanah (Tishri 1). This is determined by the day of the Molad of Tishri and the four rules of postponements (*dehiyyot*). The *dehiyyot* can postpone Tishri 1 to one or two days after the *molad*.

It is traditional to denote the Molad of Tishri by four numbers: the number of weeks elapsed since the creation, the day of the week (Sunday is day 1), the hour of the day (from 6 P.M.) and the number of halakim after the start of that hour. The useful periods of time in this convention are shown in Table 15.8. The number of weeks can usually be ignored.

Table 15.8 Periods corresponding to different numbers of lunations

Lunations	Weeks-Days-Hours-Halakim
1	4-1-12-0793
12	50-4-08-0876
13	54-5-21-0589
235	991-2-16-0595

The day and time of the Molad of Tishri of any given year can be found by adding the time of the first epochal *molad* to the times of all the lunations that precede it. The number of weeks in these calculations may be ignored, and if the number of days exceeds 7, it may be reduced by 7.

Example 1. Find the weekday and time of the Molad of Tishri of Anno Mundi (A.M.) 2. A.M. 1 is a common year with 12 lunations. Thus the time of the *Molad* of 2 A.M is found by adding the time for these 12 lunations to the time of the first epochal *Molad*.

	Days-Hours-Halakim
epochal <i>molad</i>	2-05-0204
+12 lunations	4-08-0876
<i>Molad</i> of Tishri, A.M. 2	6-14-0000

The molad of A.M. 2 fell on day 6 at 14 hr 0 hk; this corresponds to Friday at A.M. 8

Example 2. Find the day Tishri 1, A.M. 5760.

The Molad of Tishri of A.M. 5760 is preceded by 5759 complete years since the initial epoch; these comprise 303 complete nineteen-year cycles plus two common years. The lunation constants given previously may be added to the epochal *molad* to give the time and weekday of the *molad*.

		Days-Hours-Halakim
Epochal molad		2-05-0204
19-year cycles	$303 \times 2-16-0595$	2-22-1005
Ordinary years	$2 \times 4-08-0876$	1-17-0672
Leap years	$0 \times 5-21-0589$	0-00-0000
Molad of Tishri A.M. 5760		6-21-0801

Since the *molad* occurs on day 6, *dehiyyah* (4) causes Tishri 1 to be postponed to day 7. Tishri 1 of A.M. 5760 fell on day 7 (Saturday).

15.5.5 Determining the Length of the Year

The number of days in a year, Y, may be found by calculating the dates of its *molad* and that of the succeeding year, Y+1. After application of the rules of postponement, the number of

days in Y may be calculated. This should fall within the limits given in Table 15.6. It is then a simple matter to ascertain the date of the start of each month given the data in Table 15.7 or Table 15.15.

15.5.6 Terminology of the Jewish Calendar

The following are terms used in the Jewish calendar:

Deficient (*hasher*) month: a month comprising 29 days.

Full (*male*) month: a month comprising 30 days.

Common year: a year comprising 12 months, with a total of 353, 354, or 355 days.

Leap year: a year comprising 13 months, with a total of 383, 384, or 385 days.

Abundant year (*shelemah*): a year in which the months of Heshvan and Kislev both contain 30 days.

Deficient year (*haser*): a year in which the months of Heshvan and Kislev both contain 29 days.

Regular year (*kesidrah*): a year in which Heshvan has 29 days and Kislev has 30 days.

Halakim (singular, *helek*): “parts” of an hour; there are 1080 hk in an hour.

Molad (plural, *moladot*): “birth” of the Moon, taken to mean the time of the Sun of the notional calendrical moon.

Dehiyyah (plural, *dehiyyot*): “postponement”; a rule delaying 1 Tishri until after the *molad*

15.6 The Islamic Calendar

15.6.1 Introduction

The Islamic calendar is a purely lunar calendar in which months correspond to the lunar phases. As a result, the cycle of twelve lunar months regresses through the seasons over a period of about 33 years. For religious purposes, Muslims begin the months with the first visibility of the lunar crescent of the new moon.

An arithmetic calendar, sometimes called the civil Islamic calendar that approximates the lunar phase cycle is often used. This is used by chronologists in studying Islamic history and as a civil calendar in some Muslim countries.

The seven-day week is observed with each day beginning at sunset. Weekdays are specified by number, with day 1 corresponding to Sunday (see § 15.1.9). Day 6, which is called *Jum'a*, is the day for congregational prayers, but unlike the Sabbath days of the Christians and Jews, it is not a day of rest. It begins at sunset on Thursday and ends at sunset on Friday.

For religious purposes, each month begins, in principle, with the first sighting of the lunar crescent after the new moon. This is particularly important for establishing the beginning and end of Ramadan. If the weather causes uncertainties, a new month may be declared thirty days after the beginning of the preceding month. Although various predictive procedures have been used for determining first visibility (see § 15.1.5), they have always had an equivocal status. In practice, there is disagreement among countries, religious leaders, and scientists about whether to rely on observations, which are subject to error, or to use calculations, which may

Table 15.9 Months of the formal Islamic calendar

Month ^a		Month	
1.	Muharram 30	7.	Rajab 30
2.	Safar 29	8.	Shaabán 29
3.	Rabi' I 30	9.	Ramadân ^b 30
4.	Rabi' II 29	10.	Shawwál 29
5.	Jumada I 30	11.	Dhú'l-Qa'da 30
6.	Jumada II 29	12.	Dhú'l-Hijjab 29 ^c

^a Muharram, Rajab, Dhú'l-Qa'da, and Dhú'l-Hijjab are holy months

^b Month of fasting

^c In a leap year, Dhú'l-Hijjab has 30 days.

be based on poor models. It may be noted that the instant of observation of a new moon may fall in different days in different parts of the world. See URL[10] for details of the ongoing Moon watch project.

The years are reckoned from the Era of the Hijra, commemorating the migration of the Prophet and his followers from Mecca to Medina.

The history and construction of the Islamic calendar are discussed by Burnaby (1901), Mayr and Spuler (1961), and by Freeman-Grenville (1963).

15.6.2 History of the Islamic Calendar

The form of the Islamic calendar, as a lunar calendar without intercalation, was laid down by the Prophet in the Qur'an (Sura IX, verse 36–37) and in his sermon at the Farewell Pilgrimage. This was a departure from the luni-solar calendar commonly used in the Arab world, in which an intercalary month was added as deemed necessary.

Caliph 'Umar I is credited with establishing the Hijra Era in A.H. 17, but it is not known how the initial date was determined. Calculations show that the astronomical new moon (i.e., the conjunction of the Sun and Moon) occurred on +622 July 14 at 0444 UT.

15.6.3 Rules for the Arithmetic Islamic Calendar

Each year has twelve months; odd-numbered months have thirty days and even-numbered months have twenty-nine days as indicated in Table 15.9. In leap years, the twelfth month, Dhú'l-Hijjab, receives an extra day. The first day of the year is Muharram 1.

There are eleven leap years in a cycle of thirty years. These are years 2, 5, 7, 10, 13, 16, 18, 21, 24, 26, and 29 of the cycle (though a different set is sometimes used). The year 1 A.H. was the first of a cycle.

The epoch, 1/1/1 A.H. (Anno Higeræ) of the formal calendar is generally taken to correspond to Friday, 622 July 16 in the Julian calendar or Julian Day Number 1 948 440 but it started at sunset on the previous Thursday. Some authors take the epoch to correspond to this Thursday.

The mean length of a month in the thirty-year cycle is about 29.530 56 days; the calendar months will remain on average in synchrony with the lunations of the Moon to within a day for about 3000 years.

15.7 The Indian Calendars

15.7.1 History of Indian Calendars

Owing to the continuity of Indian civilisation and the diversity of cultural influences, the history of calendars in India is complex. In the mid-1950s, the Calendar Reform Committee (1955), set up by the Indian Government after independence, reported that there were about 30 calendars in use for regulating Hindu, Buddhist, and Jain religious festivals. Some of these were also used for civil dating. These calendars fall into two main types: the, so-called solar calendars (see § 15.7.2) and the luni-solar calendars (see § 15.7.3). Each has local variations which follow long-established customs and the astronomical practices of local calendar makers. In addition, Muslims in India used the Islamic calendar, and the Indian government uses the Gregorian calendar for administrative purposes.

Early allusions to a luni-solar calendar with intercalated months are found in the hymns from the Rig Veda, dating from the second millennium B.C.. Literature from 1300 B.C. to A.D. 300 provides information of a more specific nature. They describe a five-year luni-solar calendar coordinating solar years with synodic and sidereal lunar months.

Indian astronomy developed considerably during the first few centuries A.D., as advances in Babylonian and Greek astronomy reached India. Traditional calendric practices were adapted to new astronomical constants and models for the motion of the Moon and Sun. These were described in astronomical treatises of this period known as *Siddhantas* but some of these have not survived. The *Surya Siddhanta*, originated in the fourth century but was revised over the following centuries. It influenced Indian calendrics up to, and even after, the calendar reform of A.D. 1957.

The Calendar Reform Committee (1955) inaugurated, for civil use, an arithmetic solar calendar, the Reformed Saka calendar (see § 15.7.4). As well as this, it also attempted to reconcile traditional calendrical practices with modern astronomical concepts. According to their proposals, precession is accounted for and calculations of solar and lunar position are based on accurate modern methods. All astronomical calculations are performed with respect to a Central Station at longitude $82^{\circ} 30'$ East, latitude $23^{\circ} 11'$ North. For religious purposes solar days are reckoned from sunrise to sunrise.

The Committee thus set guidelines for religious calendars, which require calculations of the motions of the Sun and Moon. Tabulations of the religious holidays are prepared by the India Meteorological Department and published annually in *The Indian Astronomical Ephemeris*. Despite the attempt to establish a unified calendar for all of India, many local variations still exist and many local calendar makers continue to use traditional astronomical concepts and formulae, some of which date back 1500 years.

The day begins at sunrise, but the Indian calendars are perhaps unique in using a unit of time that may be shorter than a day: the *tithi*. Most holidays occur on specified *tithis*. Lunations are divided into 30 lunar days called *tithis*. A *tithi* is defined as the time required for the longitude of the Moon to increase by 12° over the longitude of the Sun. Thus, the length of a *tithis* may vary from about 20 hours to nearly 27 hours. During the waxing phases, *tithis* are counted from 1 to 15 with the designation *Sukla*. *Tithis* for the waning phases are designated *Krsna* and are again counted from 1 to 15. Each day is assigned the number of the *tithis* current at sunrise. Occasionally a short *tithi* will begin after sunrise and be completed before the next sunrise. Similarly a long *tithi* may span two sunrises. In the former case, a

Table 15.10 Months of the Indian solar calendar

	<i>Sankrânti</i> name	Zodiacal name	Length in days		<i>Sankrânti</i> name	Zodiacal name	Length in days
1.	Mesha	Aries	30.9	7.	Tula	Libra	29.9
2.	Vrishabha	Taurus	31.4	8.	Vrischika	Scorpio	29.5
3.	Mithuna	Gemini	31.6	9.	Dhanu	Sagittarius	29.4
4.	Karka	Cancer	31.5	10.	Makara	Capricorn	29.5
5.	Simha	Leo	31.0	11.	Kumbha	Aquarius	29.8
6.	Kanya	Virgo	30.5	12.	Mina	Pisces	30.3

number is omitted from the day count. In the latter, a day number is repeated. The logic of Days and *tithis* is analogous to that of lunar and solar months (see § 15.7.3).

The seven-day week has been used in India since at least 484 and the names of the days are derived from planetary Gods as in the Roman astrological week.

Different modes of counting the years have been used. Some initialise the count at the start of a dynasty of rulers. One such is the Saka era commencing in the year 78. Other counts are astronomically based such as the Kali-yoga whose epoch in -3101 is the date of the most recent mean conjunction of the seven planets or wandering stars (which includes the Sun and the Moon) known to the ancients.

Pingree (1978) provides a survey of the development of mathematical astronomy in India. Although he does not deal explicitly with calendars, this material is necessary for a full understanding of the history of India's calendars. Sewell (1912, 1989) and Sewell and Dikshit (1911) provide a comprehensive account of the traditional calendars. They are also discussed by Chatterjee (1987).

15.7.2 The Traditional Indian Solar Calendar

The starts of the solar months are determined by the arrival of the Sun at twelve longitudinal positions in the ecliptic measured from Aries and spaced at equal angles of 30° ; these are called *sankrântis* and are analogous to the signs of the Zodiac; they carry Sanskrit names mostly similar in meaning to the Western names as shown in Table 15.10. Since the velocity of the Sun around the ecliptic is not constant, the lengths of these months vary by several days.

Originally, the times at which the Sun reached each *sankrânti* were calculated on the basis of a mean Sun. More recent calendars are based on the true longitudes of the Sun.

15.7.3 The Traditional Indian Luni-solar Calendar

The luni-solar calendars operate in the manner described in § 15.1.11. The start of a month is determined by a new moon (or full moon, according to local custom); this feature maintains synchrony between the months and the lunations. The names of the months are similar to those given in Table 15.11.

The first month of the year is Chaitra (or in some regions, another month). The starts of the solar months are defined by the *sankrântis* (see Table 15.10). If two lunations commence

Table 15.11 Months in the Indian luni-solar calendar

Month	Month ^a
1. Chaitra	7. Asvina
2. Vaisakha	8. Kartika
3. Jyaistha	9. Margasira
4. Ashadha	10. Pausha
5. Sravana	11. Magha
6. Bhadrapada	12. Phalguna

^a The months in the Reformed Saka calendar have similar names but month 6 is named Bhada and month 9 is named Agrahayuna.

in the same solar month, the two bear the same name but the first is described with the prefix *adhika*, or intercalary. Years containing such a year have thirteen lunar months. This is equivalent to the intercalation of a month.

More rarely, a short solar month will occur without containing the start of a lunation. In that case, the name of the lunar month is omitted; this is equivalent to an extracalation. Such a (*ksaya*) month can occur only in the months near the Earth's perihelion passage. In compensation, a month in the first half of the year will have had two new moons, so the year will still have twelve lunar months. *Ksaya* months are separated by as few as nineteen years and as many as 141 years.

15.7.4 Rules of the Reformed Saka Calendar

The names of the 12 months and the number of days in each are indicated in Table 15.11. There are 365 days in each common year and 366 in a leap year.

Years are counted from the year 78 which initiates the Saka Era (E.S.), a traditional epoch in Indian calendars. The epoch of the calendar is Wednesday, 79 March 24, in the Julian calendar or Julian Day Number 1 749 995.

Leap years coincide with those of the Gregorian calendar. To determine leap years, first add 78 to the Saka year. If this sum is evenly divisible by 4, the year is a leap year, unless the sum is a multiple of 100. In the latter case, the year is not a leap year unless the sum is also a multiple of 400.

Each date in this calendar corresponds, in common years, to a corresponding date in the Gregorian calendar; thus Chaitra 1 corresponds to March 22. In leap years from Phalunga 10 to Vaisakha 21, the corresponding Gregorian date must be advanced by one day.

15.8 The Chinese Calendar

15.8.1 History of the Chinese Calendar

In China the astronomical luni-solar calendar, was a sacred document, sponsored and promulgated by the reigning monarch. For more than two millennia, a Bureau of Astronomy made astronomical observations, calculated astronomical events such as eclipses, prepared astrological predictions, and maintained the calendar (Needham 1959). A successful calendar

not only served practical needs, but also confirmed the consonance between Heaven and the imperial court.

Like the luni-solar Indian calendar (see § 15.7.2), the months of the Chinese calendar start on the day of a new moon. Solar months, analogous to the signs of the Zodiac and the Indian *sankrântis*, are used to define intercalary months, which occur every two or three years.

Analysis of surviving astronomical records inscribed on oracle bones reveals a Chinese luni-solar calendar, with intercalation of lunar months, dating back to the Shang dynasty of the fourteenth century B.C. Since then, there have been more than 50 changes in the details.

From the earliest records, the beginning of the year occurred at a new moon near the winter solstice but the choice of the month to begin the civil year varied with time and place. In the late second century B.C., a reform established the practice, which continues today, of requiring the winter solstice to occur in month 11. This reform also introduced the intercalation system in which dates of new moons are compared with the 24 solar terms (see § 15.8.3). However, calculations were based on the mean motions resulting from the cyclic relationships. Inequalities in the Moon's motions were incorporated as early as the seventh century A.D. (Sivin 1969), but the Sun's mean longitude was used for calculating the solar terms until 1644. Since 1645, the calendar is based on the true positions of the Sun and Moon and its accuracy depends on the accuracy of the astronomical theories and calculations. Before 1929, the calculation referred to observations at Beijing ($116^{\circ} 25'$ East), but since then to the 120° East meridian.

Years were counted from a succession of eras established by reigning emperors (but see § 15.8.2). Although the accession of an emperor would mark a new era, an emperor might also declare a new era at various times within his reign in an attempt to re-establish a broken connection between Heaven and Earth (as personified by the emperor). The break might be revealed by the death of an emperor, the occurrence of a natural disaster, or the failure of astronomers to predict a celestial event such as an eclipse. In the latter case, a new era might mark the introduction of new astronomical or calendrical models.

Western (pre-Copernican) astronomical theories were introduced to China by Jesuit missionaries in the seventeenth century and more modern Western concepts gradually became known. Following the revolution of 1911, the traditional practice of counting years from the accession of an emperor was abolished.

Published calendrical tables are often in disagreement about the Chinese calendar. Some of the tables are based on mean, or at least simplified, motions of the Sun and Moon. Some are calculated for other meridians than 120° East. Some incorporate a rule that the eleventh, twelfth, and first months are never followed by an intercalary month. This is sometimes not stated as a rule, but as a consequence of the rapid change in the Sun's longitude when the Earth is near perihelion. However, it is incorrect when the motions of the Sun and Moon are accurately calculated.

Reference works give a variety of rules for establishing New Year's Day and for intercalation in the luni-solar calendar. Since the calendar was originally based on the assumption that the Sun's motion was uniform through the seasons, the published rules are frequently inadequate to handle special cases.

Although the Gregorian calendar is used in the Peoples' Republic of China for administrative purposes, the traditional Chinese calendar is used for setting traditional festivals and for timing agricultural activities in the countryside. The Chinese calendar is also used by Chinese communities around the world.

Table 15.12 Celestial Stems and the Earthly Branches of the sexagenary cycle

Celestial Stems		Early Branches	
1.	jia	1.	zi rat
2.	yi	2.	chou ox
3.	bing	3.	yin tiger
4.	ding	4.	mao hare
5.	wu	5.	chen dragon
6.	ji	6.	si snake
7.	geng	7.	wu horse
8.	xin	8.	wei sheep
9.	ren	9.	shen monkey
10.	gui	10.	you fowl
		11.	xu dog
		12.	hai pig

15.8.2 Sexagenary Cycle

The sexagenary cycle is a list of 60 names, each being a combination of one of ten Celestial Stem names and one of twelve Earthly Branch names each taken in cyclic order. After six repetitions of the set of stems and five repetitions of the branches, a complete cycle of pairs is completed and a new cycle begins. The Earthly Branch names are usually translated as the names of animals, but the Celestial Stem names are said to be untranslatable. These names are shown in Table 15.12.

Sexagenary cycles were used to count months, days, and fractions of a day. The use of the sixty-day cycle is seen in the earliest astronomical records and, although it has fallen into disuse in everyday life, it is still tabulated in calendars. By contrast the sixty-year cycle was introduced in the first century A.D. or possibly a century earlier (Tung 1960; Needham 1959) and is still used today.

It is customary to number the sexagenary cycles of years so that the first cycle began in -2636 .

The initial year (*jia-zi*) of the current cycle began on 1984 February 2. *jia* is the first Celestial Stem name and *zi* is the first Earthly Branch. The numbers of the Celestial Stem (C) and of the Earthly Branch (E) of the Chinese year starting in the Gregorian year Y are given by:

$$C = 1 + \text{mod}(Y - 4, 10) \quad E = 1 + \text{mod}(Y - 4, 12) \quad (15.16)$$

$\text{mod}(A,B)$ denotes the remainder when A is divided by B. It follows that the year starting in 2000 is *geng-chen*, a year of the Dragon.

15.8.3 Major and Minor Terms

The tropical year is divided into 24 solar terms, in 15° segments of solar longitude. These divisions are paired into twelve minor terms (*Jieqi*) and twelve major terms (*Zhonggi*) as

Table 15.13 Chinese solar terms

	Term ^a	Name	Sun's Longitude	Approximate Gregorian Date	Approximate Duration in days
J1	Lichun	Beginning of Spring	315	Feb. 4	
Z1	Yushui	Rain Water	330	Feb. 19	29.8
J2	Jingzhe	Waking of Insects	345	Mar. 6	
Z2	Chunfen	Spring Equinox	0	Mar. 21	30.2
J3	Qingming	Pure Brightness	15	Apr. 5	
Z3	Guyu	Grain Rain	30	Apr. 20	30.7
J4	Lixia	Beginning of Summer	45	May 6	
Z4	Xiaoman	Grain Full	60	May 21	31.2
J5	Mangzhong	Grain in Ear	75	June 6	
Z5	Xiazhi	Summer Solstice	90	June 22	31.4
J6	Xiaoshu	Slight Heat	105	July 7	
Z6	Dashu	Great Heat	120	July 23	31.4
J7	Liqiu	Beginning of Autumn	135	Aug. 8	
Z7	Chushu	Limit of Heat	150	Aug. 23	31.1
J8	Bailu	White Dew	165	Sept. 8	
Z8	Qiufen	Autumnal Equinox	180	Sept. 23	30.7
J9	Hanlu	Cold Dew	195	Oct. 8	
Z9	Shuangjiang	Descent of Frost	210	Oct. 24	30.1
J10	Lidong	Beginning of Winter	225	Nov. 8	
Z10	Xiaoxue	Slight Snow	240	Nov. 22	29.7
J11	Daxue	Great Snow	255	Dec. 7	
Z11	Dongzhi	Winter Solstice	270	Dec. 22	29.5
J12	Xiaohan	Slight Cold	285	Jan. 6	
Z12	Dahan	Great Cold	300	Jan. 20	29.5

^aTerms are classified as minor (*Jieqi*), J, or major (*Zhongqi*), Z, followed by the number of the term.

shown in Table 15.13. These terms are numbered and assigned names that are seasonal or meteorological in meaning. For convenience the minor and major terms are denoted by J and Z, respectively, followed by the number. The major terms define the starts of solar months. Because of the ellipticity of the Earth's orbit, the lengths of the solar months vary with the season as with the Indian *Sankrântis* (see § 15.7.2).

15.8.4 Rules for the Modern Chinese Calendar

Here, we give the rules based on those that are currently used for calendars prepared by the Purple Mountain Observatory (1984) and discussed by Reingold and Dershowitz (2001) and by Aslaksen in URL[4].

The Chinese term *sui* is the period from one winter solstice (Z11) to the next.

1. The instants of conjunctions and major solar terms are calculated for meridians of 120° East.

2. The days are measured from midnight to midnight.
3. The first day of a month is the day in which a conjunction of the Moon (new moon) falls.
4. If a *sui* contains 13 complete months, one of them is a leap month.
5. This leap month is the first in the *sui* that contains no major solar term.
6. Months are assigned numbers 1 to 12 (but not names); a leap month is assigned the same number as its predecessor.
7. The winter solstice (Z11) always falls in month 11.
8. Years are counted in sexagenary year cycles.

The current sexagenary year cycle started in 1984 which is named *jia-zi*, a year of the Rat and the first day of the first month of this year was 1984 February 2 or Julian Day Number 2 445 733. Cycles are sometimes counted with the first starting in -2636 .

A corollary of rule 4 is that a *sui* with only 12 complete months contains no leap month even though one of its months may contain no major solar term.

The application of these rules results in there being about 7 leap months every 19 years. A common year of twelve months contains 353, 354, or 355 days; a leap year of thirteen months contains 383, 384, or 385.

If a month contains no solar term, the two solar terms immediately preceding its conjunction and the one following it are the same. It is convenient, here, to refer to such a month as an empty month. The number of the latest solar term preceding any event can be found by calculating the longitude of the Sun at the time of the event and rounding it down to a multiple of 30° .

The calculation of the details of the Chinese calendar for a year requires access to tables or formulae for ascertaining the instants, in local time, of the winter solstice and the lunar conjunctions at a meridian of 120° East.

15.8.5 Finding the First Day of a Chinese Year

To determine the first day of the Chinese year that starts in the Gregorian year Y :

1. Calculate the dates, W_{Y-1} and W_Y , in which the winter solstices of years $Y - 1$ and Y occur.
2. Calculate the day, M_a , of the conjunction of the Moon that falls first after W_{Y-1} and the date M_d of the conjunction that falls last on or before W_Y .
3. Calculate the number of complete lunations in the period M_a to M_d ; this is the integer, L , nearest to $(M_d - M_a)/29.53$.
4. Calculate the date of the next conjunction, M_b , after M_a .
5. If $L = 11$, there is no leap month in the *sui* W_{Y-1} to W_Y . W_{Y-1} falls in month 11 of $Y - 1$ and month 12 starts on M_a , so that the Chinese year starts with the next conjunction, M_b . Note that the Chinese year may yet contain a leap month after W_Y .
6. If $L = 12$, there is a leap month in the *sui*. This leap month is the first lunation in the *sui* that contains no major solar term. Calculate the date, M_c , of the next lunation after M_b . If the month initiated by M_a is empty, it is a leap month; likewise M_b is a leap

month if it is empty. In either case, the Chinese New Year starts on M_c , but if neither M_a nor M_b are empty, the year starts on M_b .

Example 1. Find the first day of the Chinese year starting in Gregorian year 2000.

From Eq. 15.16, we find that 2000 is *geng-chen*, a year of the Dragon.

We find the following dates:

W_{Y-1}	The winter solstice in 1999	Dec 22
W_Y	The winter solstice in 2000	Dec 21
M_a	The first conjunction after W_{Y-1}	2000 Jan 7
M_b	The next conjunction after M_a	2000 Feb 5
M_d	The last conjunction on or before W_Y	2000 Nov 25

Thus, $L = 11$, there is no leap month in the *sui* and the year begins on M_b , 2000 Feb 5.

Example 2.. Find the first day of the Chinese year in 2033 and the first day of its leap month. 2033 is an anomalous year whose details are sometimes in error in published Chinese almanacs.

We find that 2033 is *gui-chou*, a year of the Ox.

We find the following dates:

W_{Y-1}	The winter solstice in 2032	Dec 21
W_Y	The winter solstice in 2033	Dec 21
M_a	The first conjunction after W_{Y-1}	2033 Jan 1
M_b	The next conjunction after M_a	2033 Jan 31
M_c	The next conjunction after M_b	2033 Mar 1
M_d	The last conjunction on or before W_Y	2033 Dec 21

Thus, $L = 12$ and there is a leap month in the *sui*, but we find that neither M_a nor M_b are empty, so that the year begins on M_b , 2033 Jan 31.

15.8.6 Finding the First Day of Each Month in a Chinese Year

To determine which month, if any, is the leap month and to number the months of a year:

1. Determine the dates, N_Y and N_{Y+1} of the starts of the years Y and $Y + 1$ as described in § 15.8.5.
2. Also note from the calculation of N_{Y+1} which of the last two lunations of year Y that started on M_a or M_b , are empty, if either are.
3. Calculate the number of lunations, K , in the period N_Y and N_{Y+1} ; this is given by the integer nearest to $(N_{Y+1} - N_Y)/29.53$.
If $K = 12$, the year starting on N_Y has no leap months and its months are numbered 1 to 12.
If $K = 13$, one of these 13 months is a leap month. This is the month starting on M_a

or M_b if either was empty.

If neither of these is empty, find the first empty month in the first 10 months. This is the leap month.

4. Number the months from N_Y consecutively except that the leap month is assigned the same number as its predecessor.

Example 1. Find the months in the Chinese year starting in Gregorian year 2000.

We find the following:

N_Y	First day of 2000	2000 Feb 5
N_{Y+1}	First day of 2001	2001 Jan 24

Thus $K = 12$; there is no leap month and the months are numbered consecutively 1 to 12.

Example 2. Find the months in the Chinese year starting in Gregorian year 2033.

We find the following:

N_Y	First day of 2033	2033 Jan 31
N_{Y+1}	First day of 2034	2034 Feb 19
M_a	First conjunction after W_Y	2033 Dec 22
M_b	Next conjunction after M_a	2034 Jan 20

Thus $K = 13$; there is a leap month. We find that the month starting on M_a is empty so that the month starting on 2033 Dec 22 is a leap month. This leap month comes after month 11 (which contains the winter solstice) so that it is also assigned the number 11. The leap month is followed by month 12 and preceded by month 1 to 11.

15.9 The French Republican Calendar

After the French revolution, a new astronomical calendar was inaugurated in 1792. This was modelled after the Alexandrian calendar (see § 15.2.1). The year began on the day of the autumnal equinox as observed at Paris. There were 12 months in the year, each having 30 days and a meteorological name. These were followed by 5 epagomenal days with a sixth in leap years. There were three “weeks” of 10 days in each month. Each of the days of the year was given a name.

Initially there was a leap day (a *franciade*) every four years, the first in year 4 E.R. Later, modifications of this leap year rule were discussed but never ratified. The years, in the Republican Era (E.R.) were counted from the epoch on Saturday, 1792 September 22 in the Gregorian calendar or Julian Day Number 2 375 840. The calendar was not popular and was abolished by Napoleon who reinstated the Gregorian calendar in 1806.

15.10 The Bahá’i Calendar

The Bahá’i faith arose in the 19th century and its adherents use the Bahá’i or Badi calendar to determine their religious celebrations. The number 19 has special significance in this faith.

The calendar is an astronomical calendar whose year begins on the day of the vernal equinox, but commonly an arithmetic variation is used in which the year begins on March 21 in the Gregorian calendar. There are 19 months, each having 19 days, in the year followed by four epagomenal days with a fifth in leap years; these are inserted at the end of the 18th month. Each of the days of the year is given a name. The years, in the Bahá'í Era (E.B.) are counted from the epoch on Thursday, 1844 March 21 in the Gregorian calendar or Julian Day Number 2 394 647.

15.11 Calendar Conversion Algorithms

15.11.1 Introduction

The conversion of dates in one calendar to the corresponding date in another is best done by converting the date in the first calendar to a Julian Day Number and then converting that to a date in the second. In this section we provide algorithms for converting a variety of arithmetic calendars to Julian Day Numbers and vice versa. These use the method described by Hatcher (1985) and elaborated by Parisot (1986) and by Richards (1998). The calendars and the parameters that are required for each conversion are listed in Table 15.14. Conversions involving the Jewish calendar are treated in § 15.11.4. We also give algorithms for determining the day of the week and of the date of Easter Sunday.

In the algorithms given below, all variables are integers. The solidus (/) denotes integer division in which any remainder is ignored. The symbol * denotes multiplication. mod(A,B) represents the remainder when A is divided by B (i.e., A modulo B). For example:

$$\begin{array}{lll}
 0/12 = 0 & 9/12 = 0 & 15/12 = 1 \\
 0*12 = 0 & 9*12 = 108 & 15*12 = 180 \\
 \text{mod}(0,12) = 0 & \text{mod}(9,12) = 9 & \text{mod}(15,12) = 3
 \end{array}$$

Table 15.14 Selected arithmetic calendars, with parameters for algorithms

Calendar ^a	y	j	m	n	r	p	q	v	u	s	t	w	A	B	C
1 Egyptian	3968	47	0	13	1	365	0	0	1	30	0	0			
2 Ethiopian	4720	124	0	13	4	1461	0	3	1	30	0	0			
3 Coptic	4996	124	0	13	4	1461	0	3	1	30	0	0			
4 Republican ^b	6504	111	0	13	4	1461	0	3	1	30	0	0	396	578797	-51
5 Julian	4716	1401	2	12	4	1461	0	3	5	153	2	2			
6 Gregorian	4716	1401	2	12	4	1461	0	3	5	153	2	2	184	274277	-38
7 Civil Islamic	5519	7664	0	12	30	10631	14	15	100	2951	51	10			
8 Bahá'í ^c	6560	1412	19	20	4	1461	0	3	1	19	0	0	184	274273	-50
9 Saka	4794	1348	1	12	4	1461	0	3	1	31	0	0	184	274073	-36

^a Where #4, 6, 8, and 9 are calendars that intercalate leap days with the same frequency as the in the Gregorian calendar.

^b Although the French Republican calendar was originally an astronomical calendar, it was abolished in 1806 before the intercalations differed from that of the Gregorian calendar. Discussions were held to amend it with a different rule of intercalation, but the proposals were never put into effect.

^c Although the Bahá'í calendar was defined as an astronomical calendar, it is generally operated as an arithmetic calendar with a Gregorian frequency of intercalation.

Table 15.15 Number of days, $A(\mathbf{K}, \mathbf{M})$, in the Jewish calendar that precede the first day of the month \mathbf{M} for a year characterised by \mathbf{K}

\mathbf{K}	$\mathbf{M} = 1$	2	3	4	5	6	7	8	9	10	11	12	13
1	0	30	59	88	117	147	176	206	235	265	294	324	—
2	0	30	59	89	118	148	177	207	236	266	295	325	—
3	0	30	60	90	119	149	178	208	237	267	296	326	—
4	0	30	59	88	117	147	177	206	236	265	295	324	354
5	0	30	59	89	118	148	178	207	237	266	296	325	355
6	0	30	60	90	119	149	179	208	238	267	297	326	356

We also use the notation “ $A \geq B$ ” to mean “ A is greater than or equal to B ”; “ $A > B$ ” to mean “ A is greater than B ”; and “ $A < B$ ” to mean “ A is less than B .”

In several of the algorithms we use Table 15.15. The entry in this table in row \mathbf{K} and in the column headed by \mathbf{M} gives the number of days in a Jewish year which precede month \mathbf{M} , in a year of character \mathbf{K} . We refer to this as $A(\mathbf{K}, \mathbf{M})$.

A date is represented by the year, \mathbf{Y} , in the appropriate era; a month number, \mathbf{M} (epagomenal days are presumed to fall in an extra, short month), and the day of the month, \mathbf{D} . Julian Day Numbers are represented by \mathbf{J} . A number \mathbf{W} is used to represent the day of the week with $\mathbf{W} = 1$ for Sunday, . . . , $\mathbf{W} = 7$ for Saturday.

15.11.2 Calculating the Day of the Week

Algorithm 1. The day of the week number, \mathbf{W} , of Julian Day Number \mathbf{J} is given by:

$$\mathbf{W} = 1 + \text{mod}(J + 1, 7)$$

Algorithm 2. The day of the week number, \mathbf{W} , of the date $\mathbf{D}/\mathbf{M}/\mathbf{Y}$ in the Gregorian calendar is given by:

1. $a = \text{mod}(9 + \mathbf{M}, 12)$
2. $b = \mathbf{Y} - a/10$
3. $\mathbf{W} = 1 + \text{mod}(2 + \mathbf{D} + (13 * a + 2)/5 + b + b/4 - b/100 + b/400, 7)$

15.11.3 Interconverting Dates and Julian Day Numbers

Algorithm 3. To convert a date $\mathbf{D}/\mathbf{M}/\mathbf{Y}$ in one of the calendars listed in Table 15.14 to a Julian Day Number, \mathbf{J} :

1. $h = \mathbf{M} - m$
2. $g = \mathbf{Y} + y - (n - h)/n$
3. $f = \text{mod}(h - 1 + n, n)$
4. $e = (p * g + q)/r + \mathbf{D} - 1 - j$
5. $\mathbf{J} = e + (s * f + t)/u$

For the Saka calendar (10), replace step 5 by:

- 5a. $Z = f/6$
- 5b. $\mathbf{J}e + ((31 - Z) * f + 5 * Z)/u$

For Gregorian type calendars (4, 6, 8, and 9) finish with:

6. $\mathbf{J} = \mathbf{J} - (3 * ((g + A)/100))/4 - C$

Algorithm 4. To convert a Julian Day Number, \mathbf{J} , to a date in one of the calendars listed in Table 15.14:

1. $f = \mathbf{J} + j$
2. $e = r * f + v$
3. $g = \text{mod}(e, p)/r$
4. $h = u * g + w$
5. $\mathbf{D} = (\text{mod}(h, s))/u + 1$
6. $\mathbf{M} = \text{mod}(h/s + m, n) + 1$
7. $\mathbf{Y} = e/p - y + (n + m - \mathbf{M})/n$

For Gregorian type calendars (4, 6, 8, and 9) insert between steps 1 and 2:

- 1a. $f = f + (((4 * \mathbf{J} + B)/146\,097) * 3)/4 + C$

For the Saka calendar (9), replace steps 4 and 5 with:

- a. $X = g/365$
- b. $Z = g/185 - X$
- c. $s = 31 - Z$
- d. $w = -5 * Z$
- 4a. $h = u * g + w$
- 5a. $\mathbf{D} = (6 * X + \text{mod}(h, s))/u + 1$

15.11.4 Converting Dates in the Jewish Calendar

Algorithm 5. To calculate the Julian Day Number, \mathbf{J} , of the 1st day of Tishri in the Jewish year \mathbf{Y} A.M.:

1. $b = 31\,524 + 765\,433 * ((235 * \mathbf{Y} - 234)/19)$
2. $d = b/259\,20$
3. $e = \text{mod}(b, 259\,20)$
4. $f = 1 + \text{mod}(d, 7)$
5. $g = \text{mod}(7 * \mathbf{Y} + 13, 19)/12$
6. $h = \text{mod}(7 * \mathbf{Y} + 6, 19)/12$

7. If $e \geq 19440$
 or $e \geq 9924$ and $f = 3$ and $g = 0$
 or $e \geq 16\,788$ and $f = 2$ and $g = 0$ and $h = 1$
 then $d = d + 1$
8. $\mathbf{J} = d + \text{mod}(\text{mod}(d + 5, 7), 2) + 347\,997$

N.B. If integers requiring more than 15 bits are not acceptable, steps 1 to 3 may be replaced by:

- 1a. $a = (235 * \mathbf{Y} - 234)/19$
- 1b. $b = 204 + 793 * a$
- 1c. $c = 5 + 12 * a + b/1080$
- 2a. $d = 1 + 29 * a + c/24$
- 3a. $e = \text{mod}(b, 1080) + 1080 * \text{mod}(c, 24)$

Algorithm 6. To calculate the Jewish year, \mathbf{Y} , in which Julian Day Number \mathbf{J} falls:

1. $\mathbf{M} = (25\,920 * (\mathbf{J} - 347\,996))/765\,433$
 N.B. The ratio $25\,820/765\,433 = 0.033\,863\,18$;
 \mathbf{M} may be set to the integral part of $0.033\,863\,18 * (\mathbf{J} - 347\,996)$.
2. $\mathbf{Y} = 19 * (\mathbf{M}/235) + (19 * \text{mod}(\mathbf{M}, 235) - 2)/235 + 1$
3. Calculate, using algorithm 5, the Julian Day Number, \mathbf{K} , of 1 Tishri for the year \mathbf{Y} A.M.
4. If $\mathbf{K} > \mathbf{J}$: $\mathbf{Y} = \mathbf{Y} - 1$

Algorithm 7. To calculate the date in the Jewish calendar, $\mathbf{D}/\mathbf{M}/\mathbf{Y}$ which corresponds to Julian Day Number \mathbf{J} .

1. Calculate the Jewish year, \mathbf{Y} A.M., in which \mathbf{J} falls using algorithm 6.
2. Calculate, using algorithm 5, the Julian Day Number, a , of 1 Tishri 1 in the year \mathbf{Y} A.M.
3. Calculate, using algorithm 5, the Julian Day Number, b , of 1 Tishri 1 in the year $\mathbf{Y}+1$ A.M.
4. $\mathbf{K} = b - a - 352 - 27 * (\text{mod}(7 * \mathbf{Y} + 13, 19)/12)$
 N.B. \mathbf{K} characterizes the year \mathbf{Y}
5. $c = \mathbf{J} - a + 1$
6. From Table 15.15, find the highest \mathbf{M} such that $A(\mathbf{K}, \mathbf{M}) < c$
7. $\mathbf{D} = c - A(\mathbf{M}, \mathbf{K})$

Algorithm 8. To calculate the Julian Day Number, \mathbf{J} , which corresponds to a Jewish date $\mathbf{D}/\mathbf{M}/\mathbf{Y}$:

1. Calculate, using algorithm 5, the Julian Day Number, a , of 1 Tishri 1 in the year \mathbf{Y} A.M.
2. Calculate, using algorithm 5, the Julian Day Number, b , of 1 Tishri 1 in the year $\mathbf{Y}+1$ A.M.

3. $K = b - a - 352 - 27 * (\text{mod}(7 * Y + 13, 19)/12)$
4. $J = a + A(M - 1, K) + D - 1$

15.11.5 Calculating the Date of Easter

Easter Sunday in the Christian ecclesiastical calendar falls in March (month 3) or April (month 4).

Algorithm 9. To calculate the month **M** and day of the month **D** of Easter Sunday in the year **Y** in the Julian calendar:

1. $a = 22 + \text{mod}(225 - 11 * \text{mod}(Y, 19), 30)$
2. $g = a + \text{mod}(56 + 6 * Y - Y/4 - a, 7)$
3. $M = 3 + g/32$
4. $D = 1 + \text{mod}(g - 1, 31)$

Algorithm 10. To calculate the month **M** and day of the month **D** of Easter Sunday for the year **Y** in the Gregorian calendar:

1. $a = Y/100$
2. $b = a - a/4$
3. $c = \text{mod}(Y, 19)$
4. $e = \text{mod}(15 + 19 * c + b - (a - (a - 17)/25)/3, 30)$
5. $f = e - (c + 11 * e)/319$
6. $g = 22 + f + \text{mod}(140004 - Y - Y/4 + b - f, 7)$
7. $M = 3 + g/32$
8. $D = 1 + \text{mod}(g - 1, 31)$

An online calculator for calculating the date of Easter is available in URL[7].

15.12 Calendar Conversion Programs

The following programs and internet sites may be useful to the reader. The appearance of these does not constitute endorsement by the United States Department of Defense (DoD), the United States Department of the Navy, or the U.S. Naval Observatory of the linked Web sites, or the information, products or services contained therein. The above-mentioned parties do not exercise any editorial control over the information you may find at these locations nor do they accept responsibility for loss or damage arising from the use of the information on these site.

1. CALENDRICA

This interconverts between dates in 25 calendars. It is supplied with Calendrical Calculations Millennium Edition: (Reingold and Dershowitz 2001)

2. CALENDAR

This interconverts between dates in 15 calendars; it may be downloaded from URL[6]: <http://www.ricswal.plus.com>

3. CALISTO

This interconverts dates, particularly English regnal and ecclesiastical dates. It may be downloaded from URL[9].

Acknowledgments

Acknowledgments are due to LeRoy Doggett as some of the material in this chapter is taken from Chapter 12 of the previous edition (Doggett 1992).

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<http://calendopedia.com/>.
2. The home page of Dr. J.R. Stockton. It provides information with a mathematical emphasis on various aspects of calendars and astronomy:
<http://www.merlyn.demon.co.uk>.
3. A commercial site sponsoring Project Pluto, providing information on calendars:
<http://www.projectpluto.com>.
4. The home page of Professor H. Aslaksen providing information concerning several calendars including the mathematics of the Chinese calendar:
<http://www.math.nus.edu.sg/aslaksen/calendar/chinese.shtml>.
5. The Web Exhibits Project, an educational site sponsored by various educational organisations providing information on a variety of calendar:
<http://webexhibits.org>.
6. The home page of E.G. Richards containing a detailed mathematical treatment of D.A. Hatcher's method of interconverting dates. It also contains algorithms for interconverting calendars and a calendar conversion program which may be downloaded:
<http://www.ricswal.plus.com>.
7. The website of the Astronomical Applications Department of the U.S. Naval Observatory containing information on various calendars and a converter to get the Julian date to calendar date and vice versa:
<http://aa.usno.navy.mil/>.
8. A commercial site which provides links and information a variety of calendrical topics:
<http://calendarzone.com>.
9. A useful glossary of special days and calendrical terms. You can also download a calendar conversion program:
<http://homepages.tesco.net/~jk.calisto/calisto/>.
10. An invitation to participate in the Moon Watch project and report your sightings:
<http://astro.ukho.gov.uk/moonwatch/>.

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