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STATISTICAL CONSTRAINTS FOR ASTROMETRIC BINARIES WITH NONLINEAR MOTION

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ABSTRACT

Useful constraints on the orbits and mass ratios of astrometric binaries in the *Hipparcos* catalog are derived from the measured proper motion differences of *Hipparcos* and Tycho-2 ($\Delta\mu$), accelerations of proper motions ($\dot{\mu}$), and second derivatives of proper motions ($\ddot{\mu}$). It is shown how, in some cases, statistical bounds can be estimated for the masses of the secondary components. Two catalogs of astrometric binaries are generated, one of binaries with significant proper motion differences and the other of binaries with significant accelerations of their proper motions. Mathematical relations between the astrometric observables $\Delta\mu$, $\dot{\mu}$, and $\ddot{\mu}$ and the orbital elements are derived in the appendices. We find a remarkable difference between the distribution of spectral types of stars with large accelerations but small proper motion differences and that of stars with large proper motion differences but insignificant accelerations. The spectral type distribution for the former sample of binaries is the same as the general distribution of all stars in the *Hipparcos* catalog, whereas the latter sample is clearly dominated by solar-type stars, with an obvious dearth of blue stars. We point out that the latter set includes mostly binaries with long periods (longer than about 6 yr).

Key words: astrometry — binaries: general

1. INTRODUCTION

Binaries with invisible or unresolved companions may create problems in the data reduction of a large and accurate astrometric catalog. Owing to the orbital motion, which may be measurable astrometrically, these stars cannot be accurately described by the five-parameter model that includes two position components, two components of proper motion, and parallax as unknowns. Instead, a more complex model of seven or nine free parameters (including the acceleration $\dot{\mu}$ and second derivative of proper motion $\ddot{\mu}$) or a complete orbital adjustment of 12 parameters may be necessary. About 2.2% of stars in the *Hipparcos* catalog require such a special treatment (Perryman et al. 1997). With the anticipated greater accuracy of future astrometric missions such as the *Space Interferometry Mission* (*SIM*) and *Gaia*, the fraction of stars exhibiting nonlinear apparent motions will dramatically increase (Kaplan & Makarov 2003).

While astrometric ramifications of the problem have been addressed in a number of publications, less attention has been paid to the prospect of characterizing new binaries through these astrometric effects. A catalog of about 7000 $\Delta\mu$ binaries—those with significant differences in the *Hipparcos* and Fifth Fundamental Catalogue proper motions—was created by Wielen et al. (2000). Gontcharov et al. (2001) produced a similar catalog of a few hundred astrometric binaries with nonlinear motion. In further development of the approach used in these previous investigations, we derive formulae to estimate the physical size of the orbit (i.e., the semimajor axis in AU) for astrometric binaries with invisible companions from the available astrometric data in the *Hipparcos* and Tycho-2 (Høg et al. 2000) catalogs. The binaries are divided into three categories: (1) stars with statistically significant differences between the short-term *Hipparcos* proper motions and the long-term Tycho-2 proper motions; (2) stars with measured acceleration of proper motions as given in the *Hipparcos* catalog, Annex DMSA/G; and (3) stars with measured second derivatives of proper motion (besides the first derivative, or acceleration) as given in the same annex. We present two catalogs of astrometric binaries, one of which includes $\Delta\mu$ binaries, and the other $\dot{\mu}$ and $\ddot{\mu}$ binaries.

Our catalogs, which are mostly compilations of previous astrometric data, are intended for researchers interested in binary stars, in particular for the search of binary brown dwarfs and other low-mass companions. The catalogs include only reliably identified long-period astrometric binaries, which are difficult to detect by other means. The objectives of the previous Heidelberg catalogs TYC2+HIP and ARIHIP (Wielen et al. 2001a, 2001b), on the contrary, were (1) to provide the most accurate proper motions for certain samples of stars and (2) to obtain "mean" proper motions. The emphasis of the ARIHIP catalogs was heavily on the "astrometrically excellent" stars that could be used as an optical representation of the International Celestial Reference System. Our criteria are strict in the selection of high-fidelity binary stars, whereas the ARIHIP criteria select nonbinary "excellent" stars. (Essentially, for identifying binary systems, the difference is between minimizing false positives and minimizing false negatives.) Most importantly, the Heidelberg team empirically estimated an all-sky correlation between the *Hipparcos* catalog and Tycho-2 catalog proper motions. Since this is a positive correlation, applying it to the detection of stars with perfectly unperturbed motion makes it easier to achieve the statistical binarity threshold. The real correlation may differ from the mean value at different parts of the sky because of varying number density, brightness of the *Hipparcos* stars, and systematic errors (Wielen et al. 2001b). While this approach may be justified if one wants a reliable sample of single stars, the filtered perturbed stars are not necessarily binaries, and certainly not necessarily the astrometric binaries with faint companions that are the focus of this paper.

Parameters Q_0 (§ 2) and Q_1 (§ 3) are computed for all stars in our catalogs. They are related to the lower bounds of the mass ratio, and their use is exemplified in § 2 by the young binary

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TABLE 1
A Catalog of $\Delta\mu$ Binaries

HIP No. (1)	TYC1 (2)	TYC2 (3)	TYC3 (4)	*Hipparcos* $\mu_{\alpha*}$ (5)	*Hipparcos* μ_δ (6)	Tycho $\mu_{\alpha*}$ (7)	Tycho μ_δ (8)	Π (9)	$\dot{\mu}$ Binary? (10)	$\log Q_0$ (11)
68............	1178	1142	1	-99.7 ± 1.1	-315.9 ± 0.8	-69.0 ± 0.9	-305.0 ± 0.9	31.8 ± 1.2	A	-0.8
93............	4663	456	1	54.9 ± 1.3	-73.3 ± 0.7	50.8 ± 0.9	-68.7 ± 0.9	16.7 ± 1.3		-1.2
171..........	1732	2731	1	778.6 ± 2.8	-918.7 ± 1.8	829.9 ± 1.2	-989.4 ± 1.1	80.6 ± 3.0		-0.8
290..........	6989	1159	1	93.7 ± 1.1	-42.8 ± 0.6	92.5 ± 1.4	-56.7 ± 1.4	15.3 ± 1.0		-0.8
305..........	6418	1218	1	88.6 ± 1.1	-17.7 ± 0.9	100.4 ± 1.8	-20.3 ± 1.4	20.4 ± 1.0	A	-1.0
329..........	5263	680	1	143.7 ± 1.9	66.6 ± 0.8	149.9 ± 1.7	75.2 ± 1.4	9.2 ± 1.5		-0.7
356..........	4669	211	1	-42.5 ± 1.2	-17.2 ± 0.6	-33.8 ± 1.1	-20.3 ± 1.2	9.9 ± 1.1		-0.8
359..........	9346	1115	1	177.5 ± 0.9	-15.2 ± 0.7	161.6 ± 1.5	-16.5 ± 1.4	17.1 ± 0.9		-0.8
457..........	7529	512	1	-12.8 ± 0.8	-1.3 ± 0.7	-14.0 ± 0.9	3.0 ± 0.9	2.0 ± 1.0		-0.5

Notes.—Several entries are shown as an example; the complete version of the catalog is available through the CDS Web site. Col. (1): *Hipparcos* numbers. Cols. (2)–(4): Tycho identification numbers TYC1, TYC2, and TYC3. Cols. (5) and (6): *Hipparcos* proper motion components $\mu_{\alpha*}$ and μ_δ in mas yr^{-1} and their standard errors. Cols. (7) and (8): Tycho-2 proper motion components and their errors. Col. (9): *Hipparcos* parallaxes and their errors in mas. Col. (10): Flag "A" is set for stars that are also $\dot{\mu}$ binaries. Col. (11): $\log Q_0$ factors for the lower limit of q_2.

AB Dor with a brown dwarf companion. On the contrary, the "cosmic error" given in the Heidelberg compilations is a measure of astrometric disturbance and is not related directly to the orbital elements. In the appendices, we derive formulae linking the observables $\Delta\mu$, $\dot{\mu}$, and $\ddot{\mu}$ with the orbital parameters. This analysis will be useful in astrometric applications. For example, a number of *SIM* target stars and grid stars will have to be reduced with the seven-parameter astrometric model that includes acceleration components, and the formalism is directly applicable in the ensuing astrometric analysis.

2. $\Delta\mu$ BINARIES

Binarity of this type is revealed when two astrometric catalogs of proper motion are compared. Depending on the accuracy of the catalogs, a certain fraction of stars will show differences in proper motion in excess of a statistically plausible measurement error. This happens when one of the catalogs includes short-term proper motions (i.e., based on observations collected during a relatively short time), while the other is based on long-term observations of star positions. In binaries of sufficiently long periods, the reflex orbital motion of the primary will be captured in the observed short-term proper motions. The long-term proper motion, on the other hand, will be closer to the true center-of-mass motion of the system. Thus, we define $\Delta\mu$ binaries as stars that have instantaneous (or short-term) proper motions significantly different from the quasi-inertial motion of the center of mass. For the majority of stars, the unknown astrometric excursion of the photocenter from the center-of-mass position will be averaged out to quantities comparable to or smaller than the astrometric formal error. Because of the averaging effect in long-term proper motions, it is expected that mostly long-period binaries are detectable as $\Delta\mu$ binaries. Some short-period binaries can accidentally produce significant proper motion difference too if their apparent astrometric excursions are large and only a few observations are available.

Our catalog of $\Delta\mu$ binaries (Table 1) includes 1929 stars with $\Delta\mu$ differences between the *Hipparcos* and Tycho-2 catalogs larger than 3.5 σ in at least one of the coordinate components. A typical observational time span for a *Hipparcos* star is about 3.2 yr, while Tycho-2 proper motions are derived from astrometric observations often spanning over a century (Urban et al. 2000). Since *Hipparcos* proper motions were used for correcting the system of ground-based catalogs, they are positively correlated with the Tycho-2 proper motions for common stars. These correlations are not known for each star, and we chose to neglect them in order to reduce the probability of false positives, that is, to generate a more reliable set of binary stars. Because of the neglected positive correlation, the calculated $\Delta\mu$ is smaller than the true value, which tends to give an underestimate of the secondary mass. The lower bound conditions given below in this section are not violated. Stars flagged in *Hipparcos* with "C" (resolved visual binaries) or "O" (previously known astrometric binaries with orbital fits) are not included in our catalog, but a small number of stochastic solutions (flag "X") may appear there. The nearest astrometric binaries of long periods are of special interest, since they may have substellar or planetary companions.

Equation (A13), shown in Appendix A, can be rewritten in terms of the orbital period P,

$$\Delta\mu \leq \frac{2\pi \Pi R_0 M_2}{M_{\rm tot}^{2/3} P^{1/3}}, \quad (1)$$

where Π is the parallax, M_2 is the secondary mass, and $M_{\rm tot}$ is the total mass. From this, given a period estimate, a lower estimate for the secondary mass can be derived. The upper bound for $\Delta\mu$ is proportional to $q_2 = M_2/M_{\rm tot}$ and has a weak dependence on $M_{\rm tot}$, in that $\Delta\mu_{\rm max} \propto q_2 M_{\rm tot}^{1/3}$. The inequality becomes an exact equation for face-on orbits. The projected $\Delta\mu$ of an inclined orbit is always smaller than the true center-of-mass deviation of proper motion; hence, generally, heavier components are required to generate the same astrometric effect.

Another kind of uncertainty is related to the fact that the above formula is valid for the difference between instantaneous proper motion and the true center-of-mass proper motion, whereas the proper motions in the catalogs are some average of proper motions, over about 3.2 yr for *Hipparcos* and several decades to a century for Tycho-2. If the period of a system is longer than a few decades, the Tycho-2 proper motion may be very different from the true proper motion. On the other hand, if the period is shorter than several years, the *Hipparcos* proper motion is an average of instantaneous proper motions at individual times of observation. Such an average will be between the maximum and the minimum instantaneous $\Delta\mu$-values, as described by the R_0 (orbital phase) factor (Fig. 1). The averaging effect of the finite duration of *Hipparcos* measurements can be imagined as a sliding mean of

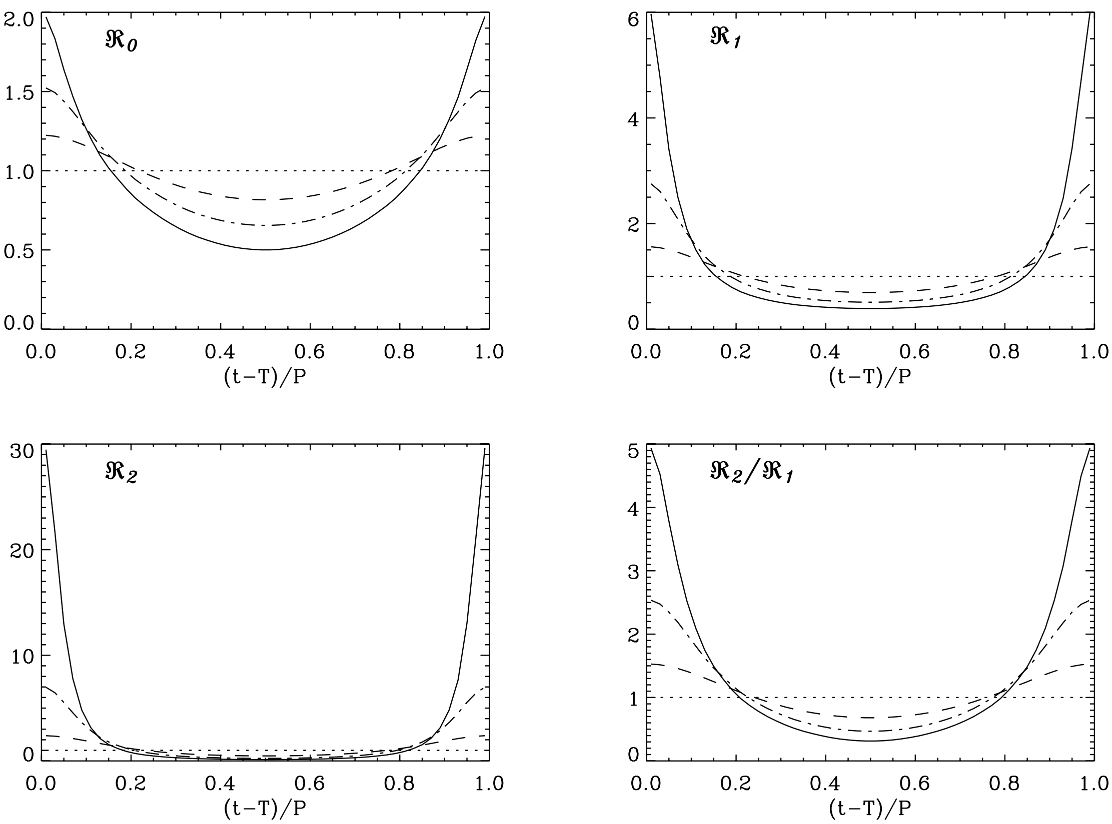

FIG. 1.—Factors R_0, R_1, R_2, and R_2/R_1 as functions of orbital phase $(t - T)/P$ for eccentricities $e = 0$ (*dotted lines*), $e = 0.2$ (*dashed lines*), $e = 0.4$ (*dash-dotted lines*), and $e = 0.6$ (*solid lines*).

the curves in Figure 1. For eccentric orbits, instantaneous proper motion deviations are greater at times of periastron passages; therefore, astrometric binaries of high eccentricity can be better detected with this method. The averaging effect will diminish the advantage of eccentric binaries, as the R_0 curves will become smoother and closer to unity. Thus, no significant overrepresentation of eccentric $\Delta\mu$ binaries is expected in the sample.

For very long period systems, the Tycho-2 and *Hipparcos* proper motions will yield a $\Delta\mu$ approaching 0 as P increases beyond several centuries. This case was analyzed in Kaplan & Makarov (2003), and it was shown that the change in measured proper motion is linear with the observational time span. The $\Delta\mu$ method will clearly miss those systems, as the apparent acceleration will be too small to produce $\Delta\mu$ larger than 2–3 mas yr^{-1}. Perhaps the next-generation astrometric satellites (*SIM* and *Gaia*) will detect many of those systems at the microarcsecond level of precision as slowly accelerating stars.

Another limitation of the astrometric method is the implicit assumption that the secondary component is much dimmer than the primary and therefore that the observable photocenter coincides with the primary, as discussed in the appendices. In some cases, when the brightness ratio can be determined photometrically or spectroscopically, appropriate corrections can be easily introduced into the formulae relating the astrometric observables and the orbital elements. In the marginal case in which both components have the same brightness, the photocenter will show no orbital motion. Most companions, however, have significantly smaller masses than the primaries, and since the intensity of light from a solar-type star with $M \in [0.5,\ 2.0]$ is approximately proportional to $M^{5.58}$ (Henry & MacCarthy 1993), the vast majority of main-sequence pairs have small photocentric effects. The inequalities given in this section and throughout the paper are valid for binaries with considerable photocentric displacements, because they specify the *lower* bounds of the apparent orbit size and motion and the secondary mass ratio.

Equation (1) shows that nearby ($\Pi > 40$ mas) solar-type stars with low-mass companions ($q_2 = M_2/M_{\rm tot}$ as low as 0.05–0.1) can be detected as $\Delta\mu$ binaries. This allows the possibility of discovering new brown dwarfs around nearby stars with orbital periods larger than 4 yr, still a poorly known domain of substellar companions (Liu et al. 2002). It may ultimately reveal whether the "brown dwarf desert," the apparent paucity of brown dwarfs with short-period orbits around solar-type stars, extends into the domain of longer periods. Companions similar to ice giant planets can be detected around the nearest and the smallest stars, such as M dwarfs.

Equation (1) is readily transformed into

$$q_2 \geq \frac{Q_0}{R_0}\left(\frac{P}{M_{\rm tot}}\right)^{1/3}, \qquad (2)$$

where $Q_0 = \Delta\mu/(2\pi\Pi)$. Here the factor Q_0 contains the two observables, while the remainder of the variables are unknown but often subject to reasonable estimates or can be bounded. Decimal logarithms of Q_0 are listed in the catalog of $\Delta\mu$ binaries (Table 1). Whenever the parallax was less than 2 mas, we assumed $\Pi = 2$ mas and gave an uncertainty flag "U" in the end of the record. Those distant stars, interestingly, have large lower bounds of q_2, implying either that the dark companion is actually more massive than the primary or that the stars are quite eccentric, and that the observation took place near the periastron time. In each particular case of interest, a more careful study should be done, since other occasional perturbations cannot be precluded. For example, a small fraction of distant stars may be unresolved visual binaries with a slowly variable companion. This may cause the photocenter to shift slowly along the line connecting the components (the effect known as a variability-imposed mover), producing apparent proper motion or even acceleration. The majority of objects in the catalog are genuine binaries, in some cases with low-mass, dim companions.

Consider, as an example, the star AB Dor (HIP 25647). It is one of the very young and active stars in the immediate solar neighborhood, whose origin is unknown (Makarov 2003). Guirado et al. (1997) combined VLBI and *Hipparcos* data to derive a putative orbit for this astrometric binary. According to their estimation, the orbital period is between 6.5 and 27.5 yr. From the catalog of $\Delta\mu$ binaries, we have $\log Q_0 = -1.3$. Assuming a total mass of 0.86 $M_\odot$, we obtain $q_2 \geq 0.10/R_0$ at $P = 6.5$, and $q_2 \geq 0.16/R_0$ at $P = 27.5$ yr. The analysis of Guirado et al. implies that the *Hipparcos* observations were taken around the periastron time, and the eccentricity is between 0.28 and 0.78. Hence, the orbital phase factor R_0 can be between 1.3 and 2.8. Taking this factor into account reverses the boundaries of the possible interval, so that $q_2 \geq 0.07$ at $P = 6.5$, and $q_2 \geq 0.06$ at $P = 27.5$ yr. We finally conclude that the secondary mass in this system can be as small as 0.05–0.06 $M_\odot$, in good agreement with the estimates in Guirado et al. (1997) of 0.08–0.11. The actual mass may indeed be somewhat larger than our lower limits, since the inclination of the orbit i is probably about 60°.

3. $\dot{\mu}$ BINARIES AND $\ddot{\mu}$ BINARIES

All currently known $\dot{\mu}$ and $\ddot{\mu}$ binaries are collected in a catalog of 2622 stars (Table 2). The data are mostly copied from the Double and Multiple Star Annex, Part G, of the *Hipparcos* catalog. We added only cross-reference flags "M" for stars that are also $\Delta\mu$ binaries, estimates related to the mass ratio lower bounds, and uncertainty flags "U" for the latter whenever the *Hipparcos* parallax was less than 2 mas. It is noted again that the orbital estimates (Appendix B) suffer three types of uncertainties: (1) the projection effect for inclined orbits, (2) the averaging effect due to the finite duration of the observation, and (3) the orbital phase effect for eccentric orbits. The former effect always makes the observed accelerations smaller than the true orbital accelerations, and thus our upper estimates of a larger. The latter two effects may act either way. In order to better illustrate these effects, the orbital phase factors R_0, R_1, and R_2 are depicted in Figure 1 as functions of phase for $e = 0$, 0.2, 0.4, and 0.6. The heights of the peaks near the periastron increase for accelerating binaries and stars with significant second derivatives; hence, the effect of eccentricity becomes stronger. Although the duration of enhanced nonlinear motion around the periastron time is only about 1/10 of the orbital period, eccentric binaries that happen to be observed close to their periastron have much better chances of being detected. Indeed, the magnitudes of $\dot{\mu}$ and $\ddot{\mu}$ are proportional to the parallax Π (eqs. [B12] and [B17]), while the number of detectable eccentric binaries grows as Π^{-3} in the near-solar neighborhood. Thus, binaries of extreme eccentricity could be detected even at relatively large distances.

Equation (B14) from Appendix B can be rewritten in terms of the orbital period P:

$$\dot{\mu} \leq \frac{(2\pi R_1)^2 \Pi M_2}{M_{\rm tot}^{2/3} P^{4/3}}. \qquad (3)$$

From this, given a period estimate, a lower estimate for the secondary mass can be derived. Just as for $\Delta\mu$, the upper bound for $\dot{\mu}$ is proportional to $q_2 = M_2/M_{\rm tot}$ and has a weak dependence on $M_{\rm tot}$, in that $\dot{\mu}_{\rm max} \propto q_2 M_{\rm tot}^{1/3}$. The inequality becomes an exact equation for face-on orbits. Equation (3) is readily transformed into

$$q_2 \geq \frac{Q_1}{R_1^2} \frac{P^{4/3}}{M_{\rm tot}^{1/3}}, \qquad (4)$$

where $Q_1 = \dot{\mu}/(4\pi^2 \Pi)$. Decimal logarithms of Q_1 are listed in the catalog of $\dot{\mu}$ binaries (Table 2). Whenever the parallax was below 2 mas, we assumed $\Pi = 2$ mas and gave an uncertainty flag "U" in the end of the record.

Figure 1 (*bottom right*) shows why second derivatives of proper motions can be larger in magnitude than accelerations, even in long-period binaries. This is caused by the large curvature of highly eccentric orbits near the periastron. Since the ratio R_2/R_1 is significantly larger than unity only within $\pm 0.1P$, the probability of observing such accelerations is fairly small for a given eccentric binary. Presumably, very eccentric binaries are rare, too. The fraction of stars with $\ddot{\mu}/\dot{\mu} > 1$ should be quite small in an unbiased sample. However, most of the stars with significant accelerations and second derivatives have $\ddot{\mu}/\dot{\mu} > 1$, which is probably a statistical bias due to the much higher detection threshold for $\ddot{\mu}$. Stars with significant $\ddot{\mu}$ can hardly have periods $P \gg 1$ yr, as discussed in § 4.

The statistical significances of accelerations and second derivatives are given as statistics in columns (7) and (10) of Table 2, respectively. These statistics are equivalents to χ^2 statistics with 2 degrees of freedom for the case of a multivariate normal distribution. Estimated accelerations were considered statistically significant when that quantity was larger than 3.44, and similarly for second derivatives of proper motion. This criterion corresponds to the 3 σ level of significance of a one-dimensional Gaussian distribution, or a 0.27% probability of a null hypothesis (that the true parameter value is 0).

4. OVERVIEW OF ASTROMETRIC BINARIES

From relations (A13) and (B12), one obtains

$$\frac{\Delta\mu}{\dot{\mu}} \approx \frac{P}{2\pi} (1 + e\cos E)^{1/2} (1 - e\cos E)^{3/2}. \qquad (5)$$

This equation is strict for face-on orbits and a good approximation for moderately inclined orbits. For circular orbits, the ratio of instantaneous proper motion difference and acceleration is roughly $P/2\pi$. This approximate equality is expected to hold for the majority of astrometric binaries of moderate eccentricities, since the probability of catching a star near its periastron is small. However, very eccentric binaries may tip the balance toward stars with small $\Delta\mu/\dot{\mu}$. With that in mind, we discriminate in the set of astrometric binaries in the two catalogs a set of binaries with significant $\Delta\mu$ but nonmeasurable $\dot{\mu}$, and a set of binaries with significant $\dot{\mu}$ but insignificant $\Delta\mu$. It is expected that most of the former should have periods longer than about 6 yr, while the latter group should include mostly stars with periods shorter than 6 yr.

Figure 2 shows distributions of $B - V$ colors of the short-period and long-period binaries, selected as described above, and the general distribution of *Hipparcos* stars (*solid line*) normalized to unit area. There is no appreciable difference between the distributions of short-period binaries and the general sample. Hence, short-period binaries have the same rate of appearance in

TABLE 2

A CATALOG OF $\dot{\mu}$ BINARIES AND $\ddot{\mu}$ BINARIES

HIP No. (1)	TYC1 (2)	TYC2 (3)	TYC3 (4)	$\dot{\mu}_{\alpha*}$ (5)	$\dot{\mu}_{\delta}$ (6)	Sig. Level (7)	$\ddot{\mu}_{\alpha*}$ (8)	$\ddot{\mu}_{\delta}$ (9)	Sig. Level (10)	Π (11)	$\Delta\mu$ Binary? (12)	$\log Q_1$ (13)	Uncertain? (14)
62	8022	411	1	-6.41 ± 2.69	-5.16 ± 1.71	3.53				5.3 ± 1.1		-1.3	
68	1178	1142	1	3.04 ± 2.50	-7.04 ± 1.66	5.33				31.8 ± 1.2	M	-2.5	
104	6989	95	1	9.75 ± 2.79	5.97 ± 1.38	5.63				8.2 ± 1.2		-1.3	
305	6418	1218	1	-4.26 ± 2.97	9.95 ± 1.40	7.11				20.4 ± 1.0	M	-2.2	
340	7529	819	1	3.16 ± 3.19	-0.99 ± 2.99	1.15	-29.08 ± 8.87	-17.63 ± 6.97	3.84	14.9 ± 1.0		-2.4	
493	1181	1071	1	-0.60 ± 1.63	3.96 ± 1.34	3.58				26.2 ± 0.8		-2.5	
611	7526	641	1	-17.65 ± 3.33	-2.38 ± 3.24	5.42	19.19 ± 9.25	-25.18 ± 7.91	4.15	7.2 ± 1.1		-1.2	
646	5841	423	1	-7.51 ± 2.53	-2.83 ± 1.02	4.02				3.6 ± 0.9		-1.1	
648	9350	1236	1	6.77 ± 1.80	3.36 ± 1.52	4.19	-17.15 ± 4.31	9.84 ± 4.40	4.96	8.9 ± 0.6		-1.5	
695	4018	3889	1	0.18 ± 1.13	6.17 ± 1.66	3.73				2.2 ± 0.6		-1.1	
741	6995	264	1	11.84 ± 3.24	5.29 ± 2.38	4.07				0.1 ± 1.4		-0.4	U

NOTES.—Several entries are shown as an example; the complete version of the catalog is available through the CDS Web site. Col. (1): *Hipparcos* numbers. Cols. (2)–(4): Tycho identification numbers TYC1, TYC2, and TYC3. Cols. (5) and (6): *Hipparcos* acceleration components $\dot{\mu}_{\alpha*}$ and $\dot{\mu}_{\delta}$ and their errors in mas yr^{-2}. Col. (7): Significance levels of acceleration terms. Cols. (8) and (9): Second derivatives of proper motion $\ddot{\mu}_{\alpha*}$ and $\ddot{\mu}_{\delta}$ and their errors in mas yr^{-3}. Col. (10): Significance levels of second derivatives. Col. (11): *Hipparcos* parallaxes and their errors in mas. Col. (12): Flag "M" is set for stars that are also $\Delta\mu$ binaries. Col. (13): $\log Q_1$ factors for the lower limits of q_2. Col. (14): Flag "U" for uncertain $\log Q_1$ estimates for stars with small or negative parallaxes.

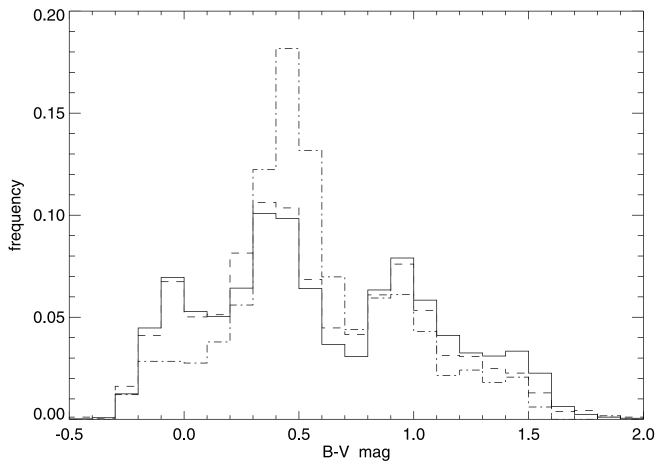

FIG. 2.—Distributions of $B - V$ colors for all 118,218 *Hipparcos* stars (*solid line*), astrometric binaries with significant $\Delta\mu$ differences but insignificant accelerations (*dot-dashed line*; 1161 stars), and with significant accelerations but insignificant $\Delta\mu$ (*dashed line*; 1854 stars).

all spectral types. This is obviously not the case with long-period astrometric binaries, which tend to appear mostly as solar-type spectral types (late F and G) and rarely as B- and A-type stars. It may be tempting to interpret this result in terms of different binary formation mechanisms (e.g., fragmentation vs. capture) producing distinct period distributions for more massive stars. We give a cautionary note that early-type stars are also younger, and some evolutionary effects cannot be precluded.

5. SUMMARY

The two catalogs of astrometric binaries presented here, based on the results of the *Hipparcos* mission and older ground-based astrometric catalogs, can be useful in the ongoing quest for low-mass binaries and brown dwarfs in the solar neighborhood. The method of astrometric motion analysis is sensitive in the most difficult area of orbital parameters for both spectroscopic and imaging investigation, i.e., orbital periods between 3 and 100 yr. Although the proper motion data are not sufficient to estimate the orbital elements, approximate constraints are useful for statistical studies and for selecting promising candidates among nearby stars. In most cases, additional information via, e.g., radial velocity monitoring or new astrometric measurements will be required to prove the existence of a low-mass companion and to estimate its mass. The nearest binaries are especially interesting, since the accuracy of the *Hipparcos* and Tycho-2 data is sufficient to detect giant planets. However, care should be exercised with nearby stars that have large radial velocities, since their large parallaxes and considerable secular accelerations were not taken into account in deriving the Tycho-2 proper motions. For example, existence of planetary companions to Barnard's star and Kapteyn's star should not be taken for granted just because these stars are included in the catalog of astrometric binaries. Barnard's star, in particular, has a large secular acceleration of 1.24 mas yr^{-2} in declination that amounts to an astrometric offset of $6''.2$ over 100 yr. This offset may account for the $\approx$50 mas yr^{-1} difference between the Tycho-2 and *Hipparcos* proper motions. Such objects require more sophisticated astrometric analysis.

Because of the limited accuracy of the available astrometric data, the number of astrometric binaries is currently not large. However, important conclusions can be drawn from the available sample, especially on a well-defined volume-limited sample of stars in which a reasonable degree of completeness can be anticipated. Some aspects of binary formation at low q_2 can be addressed, such as distributions of orbital elements and their dependencies on age and metallicity.

The fraction of astrometric binaries will dramatically increase when the next-generation astrometry projects provide us with much more accurate data (*Gaia* and *SIM*). In fact, astrometric binaries may prove to be a considerable difficulty in the complicated data reduction systems for these missions. If, for example, brown dwarf companions are widespread in long-period binaries, a large number of reference stars may be completely unsuitable for processing with the standard astrometric model of linear motion and parallax.

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APPENDIX A

ORBIT CONSTRAINTS FROM $\Delta\mu$

The apparent motion of a binary in the plane of celestial projection is described by (Heintz 1978)

$$x = A(\cos E - e) + F\sqrt{1-e^2}\sin E, \qquad \text{(A1)}$$

$$y = B(\cos E - e) + G\sqrt{1-e^2}\sin E, \qquad \text{(A2)}$$

where x and y are the coordinates, e is the eccentricity of the orbit, and E is the eccentric anomaly related to the mean anomaly M by Kepler's equation

$$M = 2\pi\frac{t-T}{P} = E - e\sin E. \qquad \text{(A3)}$$

As usual, T is the periastron epoch and P is the orbital period. The Thiele-Innes constants are related to the remaining orbital elements by

$$A = a_a(\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i), \qquad \text{(A4)}$$

$$B = a_a(\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i), \qquad \text{(A5)}$$

$$F = a_a(-\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i), \qquad \text{(A6)}$$

$$G = a_a(-\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i), \qquad \text{(A7)}$$

where a_a is the astrometric semimajor axis (whether for the primary, secondary, or the photocenter), ω is the longitude of the periastron in the plane of orbit, and Ω is the position angle of the node in the plane of projection.

The first derivative of E is

$$\dot{E} = \frac{2\pi}{P}\frac{1}{1-e\cos E}. \qquad \text{(A8)}$$

Differentiating equation (A2) with respect to time obtains (see also Brouwer & Clemence 1961)

$$\dot{x} = \left(-A\sin E + F\sqrt{1-e^2}\cos E\right)\dot{E}, \qquad \text{(A9)}$$

$$\dot{y} = \left(-B\sin E + G\sqrt{1-e^2}\cos E\right)\dot{E}. \qquad \text{(A10)}$$

The square of the apparent instantaneous orbital velocity is

$$\dot{x}^2 + \dot{y}^2 = a_a^2 \left[\left(\cos\omega \sin E + \sin\omega \sqrt{1-e^2} \cos E \right)^2 + \left(\sin\omega \sin E - \cos\omega \sqrt{1-e^2} \cos E \right)^2 \cos^2 i \right] \dot{E}^2. \tag{A11}$$

Note that the magnitude of the instantaneous velocity is independent of Ω. Since $\cos^2 i \leq 1$, and for the primary component

$$a_a = a_1 \Pi = \frac{a \Pi M_2}{M_{\rm tot}}, \tag{A12}$$

we obtain after replacing $\cos^2 i$ with unity,

$$\Delta\mu \leq \frac{2\pi \Pi R_0 M_2}{\sqrt{a M_{\rm tot}}}, \tag{A13}$$

where Π is the parallax in the same units as the semimajor axis of the primary a_a, a_1 is the semimajor axis of the primary in AU, M_2 is the mass of the secondary, $M_{\rm tot}$ is the total mass, $\Delta\mu$ is the astrometrically measured instantaneous orbital velocity of the primary in the plane of projection, with respect to the center of mass, and the time-dependent (orbital phase) term is

$$R_0 = \left(\frac{1 + e\cos E}{1 - e \cos E} \right)^{1/2}. \tag{A14}$$

This can be transformed into a constraint for the full semimajor axis of the orbit:

$$a \leq \frac{(2\pi)^2 \Pi^2 R_0^2 M_2^2}{M_{\rm tot} \Delta\mu^2}. \tag{A15}$$

If the share of intensity of the secondary star in the total intensity of the system, $k_2 = I_2/(I_1 + I_2)$, is not negligible and the separation between the components is much smaller than the resolution limit of the instrument, M_2 in equations (A13) and (A15) should be replaced with $(k_1 M_2 - k_2 M_1)$. In this case, the astrometric excursion of the photocenter is always smaller than the actual excursion of the primary, and subsequently, the observed $\Delta\mu$ is smaller too.

For *Hipparcos* stars that have significant differences in proper motion between the long-term Tycho-2 and Sixth Fundamental Catalogue but do not have measurable accelerations, a lower bound of a can be derived from the somewhat arbitrary condition that the orbital period should be greater than or approximately equal to 5 times the mission duration (typically, 3.2 yr). This provides a rough condition:

$$a \gtrsim 6.3 M_{\rm tot}^{1/3}. \tag{A16}$$

An alternative way to derive the equality corresponding to equation (A13) is to use the vis viva integral, representing conservation of energy,

$$v^2 = 2 G_{\rm gr} M_{\rm tot} \left(\frac{1}{r} - \frac{1}{2a} \right), \tag{A17}$$

in which r is the instantaneous distance between the two components, v is the relative speed, $G_{\rm gr}$ is the gravitational constant, and a is the semimajor axis of the relative orbit. This can be rearranged to

$$v^2 = \frac{G_{\rm gr} M_{\rm tot}}{a} \left(\frac{2a}{r} - 1 \right), \tag{A18}$$

where $r = a(1-e^2)/(1 + e\cos\theta)$ and θ is the angle at the primary between the direction to periastron and the instantaneous position of the secondary; i.e., θ is the true anomaly. In terms of eccentric anomaly, $r = a(1 - e\cos E)$. It is easy to show that the factor in parentheses above equals R_0^2.

For face-on orbits, the projection factor is 1 everywhere, and $\Delta\mu = v \Pi M_2 / M_{\rm tot}$. Therefore, we have

$$\Delta\mu = \left(\frac{G_{\rm gr} M_{\rm tot}}{a} \right)^{1/2} \frac{\Pi R_0 M_2}{M_{\rm tot}}, \tag{A19}$$

and if $G_{\rm gr} = 4\pi^2$, this becomes

$$\Delta\mu = \frac{2\pi \Pi R_0 M_2}{\sqrt{a M_{\rm tot}}}, \tag{A20}$$

the same as equation (A13) applied to face-on orbits.

APPENDIX B

ORBIT CONSTRAINTS FROM $\dot{\mu}$

Differentiating equation (A10) with respect to time obtains

$$\ddot{x} = -\left(A\cos E + F\sqrt{1-e^2} \sin E \right) \dot{E}^2 + \left(-A \sin E + F\sqrt{1-e^2} \cos E \right) \ddot{E}, \tag{B1}$$

$$\ddot{y} = -\left(B\cos E + G\sqrt{1-e^2} \sin E \right) \dot{E}^2 + \left(-B \sin E + G\sqrt{1-e^2} \cos E \right) \ddot{E}. \tag{B2}$$

Substituting

$$\dot{E}^2 = -\frac{1 - e\cos E}{e \sin E} \ddot{E}, \tag{B3}$$

squaring, and adding up equations (B1) and (B2) obtains

$$\dot{\mu}^2 = \ddot{x}^2 + \ddot{y}^2 = \frac{\ddot{E}^2}{e^2} \left(p_1 q_1^2 - 2 p_3 q_1 q_2 + p_2 q_2^2 \right), \tag{B4}$$

where

$$p_1 = A^2 + B^2 = a_a^2 \left(\cos^2\omega + \sin^2\omega \cos^2 i \right), \tag{B5}$$

$$p_2 = F^2 + G^2 = a_a^2 \left(\sin^2\omega + \cos^2\omega \cos^2 i \right), \tag{B6}$$

$$p_3 = AF + BG = a_a^2 \left(-\cos\omega \sin\omega + \cos\omega \sin\omega \cos^2 i \right), \tag{B7}$$

$$q_1 = \frac{1 - \cos E}{\sin E}, \tag{B8}$$

$$q_2 = \sqrt{1 - e^2}. \tag{B9}$$

Substituting the expressions for p_1, p_2, and p_3 leads to a quadratic form similar to equation (A11):

$$\ddot{x}^2 + \ddot{y}^2 = \frac{a_a^2}{e^2}\ddot{E}^2\Big[(\cos\omega q_1 + \sin\omega q_2)^2 + (\sin\omega q_1 - \cos\omega q_2)^2\cos^2 i\Big]. \tag{B10}$$

Since $\cos^2 i \leq 1$, using equation (A12) and

$$\ddot{E} = -\left(\frac{2\pi}{P}\right)^2 \frac{e\sin E}{(1 - e\cos E)^3}, \tag{B11}$$

we derive for the instantaneous acceleration of proper motion in the plane of projection

$$\dot{\mu} = \sqrt{\ddot{x}^2 + \ddot{y}^2} \leq \left(\frac{2\pi R_1}{a}\right)^2 \Pi M_2, \tag{B12}$$

where Π is the parallax in the same units as the astrometric excursion a_a, M_2 is the mass of the secondary component, and the time-dependent term

$$R_1 = \frac{1}{1 - e\cos E}. \tag{B13}$$

This can be transformed into a constraint for the full semimajor axis of the orbit:

$$a \leq 2\pi R_1 \sqrt{\frac{\Pi M_2}{\dot{\mu}}}. \tag{B14}$$

If the share of intensity of the secondary star in the total intensity of the system, $k_2 = I_2/(I_1 + I_2)$, is not negligible and the separation between the components is much smaller than the resolution limit of the instrument, M_2 in equations (B12) and (B14) should be replaced with $(k_1 M_2 - k_2 M_1)$. In this case, the astrometric excursion of the photocenter is always smaller than the actual excursion of the primary, and subsequently, the observed $\dot{\mu}$ is smaller too.

For *Hipparcos* stars that have significant proper motion accelerations but do not have orbital solutions, a lower bound of a can be derived from the somewhat arbitrary condition that the orbital period should be greater than or approximately equal to 2 times the mission duration (typically, 3.2 yr). This provides a rough condition:

$$a \gtrsim 3.4 M_{\rm tot}^{1/3}. \tag{B15}$$

Alternatively, the equality relation (B12), corresponding to a face-on orbit case, can be derived directly from the formula for orbital acceleration,

$$\dot{v} = \frac{G_{\rm gr} M_{\rm tot}}{r^2}, \tag{B16}$$

along the lines described at the end of Appendix A, using the relation $\dot{\mu} = \dot{v}\Pi M_2/M_{\rm tot}$ for instantaneous acceleration of the primary with respect to the center of mass.

We omit the somewhat cumbersome derivation for $\ddot{\mu}$ and give the final formula:

$$\ddot{\mu} = \sqrt{\left(\frac{\partial^3 x}{\partial t^3}\right)^2 + \left(\frac{\partial^3 y}{\partial t^3}\right)^2} \leq \frac{(2\pi)^3 \Pi M_2 \sqrt{M_{\rm tot}} R_2}{a^{7/2}}, \tag{B17}$$

where the time-dependent term

$$R_2 = \frac{\sqrt{1 + e^2(3 - 4\cos^2 E)}}{(1 - e\cos E)^4}. \tag{B18}$$

This can be transformed into a constraint for the full semimajor axis of the orbit:

$$a \leq \frac{(2\pi)^{6/7}(R_2 \Pi M_2)^{2/7} M_{\rm tot}^{1/7}}{\ddot{\mu}^{2/7}}. \tag{B19}$$

Inequalities (B17) and (B19) become exact equations for face-on orbits ($i = 0°$).

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