# PROSPECTS FOR IMPROVING THE MASSES OF MINOR PLANETS

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ABSTRACT. Among the largest uncertainties in the fundamental constants of astronomy are the masses of the minor planets. They constitute the largest source of uncertainty in the ephemerides of the inner planets. Few asteroid masses are known with an uncertainty of better than about 50%. With a few exceptions, minor planet masses are determined by observing the perturbation of a massive minor planet on a smaller one during a close encounter. The recent explosion of discoveries of minor planets and the increased accuracy of modern astrometric catalogs means that it may be possible to discover more such encounters. A series of simple filters were developed to search for these encounters. For example, three encounters were found with Ceres, which are almost certainly strong enough to provide a mass estimate with a significantly smaller uncertainty than current estimates prior to Dawn's arrival there in 2015.

## 1. INTRODUCTION

The masses of the minor planets are one of the least known of the constants of fundamental astronomy. Few minor planets have masses with reliable uncertainties less than 1% the majority have masses based on assumptions of their size deduced from photometry and chemical makeup deduced from spectroscopy. At the same time, the perturbations of the minor planets constitute the largest unsolved problem in the refinement of the ephemerides of the inner planets (Standish & Fienga 2002).

As a part of the mission planning for the Dawn Mission[1] to (1) Ceres and (4) Vesta, a study was done (Hilton 2008) to determine if there were any encounters between now and Dawn's arrival at these two minor planets that could significantly improve upon the ∼1% uncertainty of the current best estimate for their masses provided by Konopliv et al. (2006). The results of this search is the main result of this paper.

## 2. THE TECHNIQUE

A series of simple filters were devised in Hilton (2008) to find those encounters that occur at a close enough distance and slow enough velocity to produce an observable change in the orbit of the perturbed minor planet. These filters are based on the expected change in the semi-major axis of the perturbed minor planet using the scattering equation

$$\tan\frac{1}{2}\theta = \frac{G(M+m)}{v^2 b} \qquad (1)$$

where $\theta$ is the scattering angle, $G$ is the gravitational constant, $M$ and $m$ are the masses of the massive and perturbed minor planets, respectively, $v$ is the velocity of $m$ relative to $M$, and $b$ is the impact parameter[2]. For simplicity, Keplerian orbits are assumed. It is also assumed that $M \gg m$, so that the center of mass is essentially the center of mass of the massive minor planet and the mass of the perturbed minor planet can be ignored. These filters greatly constrain the search space for even the largest minor planet. Table 1 shows the parameter space in which a perturbed minor planet may be perturbed by Ceres or Vesta by enough to change the semi-major axis by $10^{-5}$ AU.

Although developed for the improvement of the masses of Ceres and Vesta, the techniques developed for finding these encounters are general and may be applied to the mass determination of any massive minor planet. All that is required for input are:

[1] http://dawn.jpl.nasa.gov/
[2] The minimum distance between $m$ and $M$ if there were no force between the two minor planets.

| | $\Delta a$ (AU) | $\Delta e$ | $\Delta\omega$ (′) | $i$ (′) |
|---|---|---|---|---|
| Ceres | 0.014 | 0.012 | 21 | 41 |
| Vesta | 0.007 | 0.009 | 16 | 31 |

Table 1: Maximum differences in orbital elements that will allow a change of at least $10^{-5}$ AU to a perturbed minor planet.

- An initial order of magnitude estimate for the mass of the massive minor planet.
- A database of candidate perturbed minor planets with elements similar to those of the massive minor planet
- A time window over which observations of the perturbed minor planet will be available.

Fulfilling these requirements is a balancing act between several competing interests:

- The size of the perturbation by the massive minor planet is quite small, even for the largest minor planet. Thus, a long time span for observations is desired to maximize the observed effect of the perturbation.
- The perturbed minor planet is subject to perturbations by many other, relatively low mass minor planets. That is, it is subject to unmodelled forces. Thus, a short series of observations is desired to minimize the effects of the unmodelled forces.
- A larger perturbed minor planet is brighter and thus its position may be directly compared to state of the art astrometric catalogs such as the UCAC (Zacharias et al. 2004). This results in a more accurate orbit allowing a smaller perturbation to be detected. Thus, just as long as its mass is negligible in comparison to the massive minor planet, a larger perturbed minor planet is desirable.
- The size distribution of minor planets is a power law with a negative index (e.g. Bottke et al. 1005). Thus, the perturbed minor planet in an encounter with the massive minor planet is more likely to be small and dim instead of large and relatively bright.

Using the currently best possible time and space resolution for an optical astrometric observation, propagation of errors shows that to determine the mass of a large minor planet with an uncertainty of 5% using a single year of observations –six months prior to the encounter and six months afterward– requires a change in the semi-major axis of the perturbed minor planet $\sim 10^{-5}$ AU (Hilton 2008). The accuracy of the mass determination is directly proportional to the length of time over which the perturbed minor planet is observed both prior to and after the encounter and the change in the perturbed minor planet's semi-major axis. It is also directly proportional to the quadrature combination of the accuracy in the astrometric position and time of observation.

The effect of these filters is to constrain the volume of orbital element space that needs to be searched to find encounters likely to lead to a reliable, high accuracy minor planet mass determination. In the cases of Ceres and Vesta this space is further reduced by requiring that the encounter occur in the period between the discovery of the perturbed minor planet and Dawn's arrival at each of these two targets.

## 3. RESULTS FOR CERES AND VESTA

Unfortunately, for Vesta only a single encounter was found that could improve its mass. This encounter, with 2004 $RO_{69}$, does not occur until June 2011, only five months prior to Dawn's arrival at Vesta. Thus, there is simply not enough time to make the observations necessary to make a mass determination using this encounter. The best estimate of Vesta's mass will remain that of Konopliv et al. (2006) until a mass can be derived from Dawn's interaction with Vesta.

On the other hand, three excellent encounters were found for improving the mass of Ceres. These encounters are detailed in Table 2. Each encounter has its own unique advantages.

*2004 $BW_{137}$*. The encounter with 2004 $BW_{137}$ is of interest because it is the earliest of the three. The current estimate of the uncertainty in the semi-major axis of 2004 $BW_{137}$ from the AstDys web site (Knežević & Milani 2003) is $5 \times 10^{-7}$ AU from 34 observations made between 2000 and 2005. If observations can be made of a similar accuracy to those prior to the encounter the potential uncertainty in the mass of Ceres derived from this encounter will be 0.2%. The next opposition of 2004 $BW_{137}$ occurs during Nov. 2007. Thus, a preliminary Ceres mass from this encounter should be available in early 2008.

| Perturbed Minor Planet | Date | $a$ (AU) | $v$ (m/s) | $\delta a$ (AU) |
|---|---|---|---|---|
| 2004 $BW_{137}$ | 2005.69 | 2.865 | 6 | $5 \times 10^{-4}$ |
| 4325 Guest | 2008.58 | 2.749 | 18 | $1 \times 10^{-4}$ |
| 2000 $EM_{61}$ | 2013.14 | 2.802 | -1 | $6 \times 10^{-2}$ |

Table 2: Encounters with Ceres prior to Dawn's arrival which have a potential for reducing the uncertainty in Ceres' mass.

*4325 Guest*. The encounter with 4325 Guest has two advantages. First, Guest was discovered in 1975 and has well over 700 observations. The current uncertainty in its semi-major axis as estimated by AstDys is $4 \times 10^{-8}$ AU, an order of magnitude smaller than that of 2004 $BW_{137}$. Probably even more important is Guest is much brighter with a mean opposition $V$ magnitude of approximately 15.7. This additional brightness means

- Guest may be observed for a longer period of time around its opposition.
- Guest may be observed by smaller telescopes than the other two test minor planets. Thus, there are more opportunities to observe it.
- Guest is bright enough that its position may be reduced using the accurate UCAC catalog.

Thus, Guest has the potential advantage of allowing more numerous and more accurate observations. As a result, it may yield a more accurate mass than 2004 $BW_{137}$ even though the expected change in Guest's semi-major axis is only 1/5 that of 2004 $BW_{137}$.

*2000 $EM_{61}$*. The encounter of 2000 $EM_{61}$ occurs in late January-early February 2013, just two years before Dawn's arrival at Ceres. However, the strength of the encounter is enormous with an estimated $\delta a$ of $6 \times 10^{-2}$ AU. The strength of the encounter arises from the extremely slow speed of 1 m s$^{-1}$. The most likely source of error in the size of the perturbation is thus an error in the encounter speed. Hilton (2008) shows that, assuming a Keplerian orbit, the value of $\delta a$ depends on the encounter speed to the fifth power. Thus, if the encounter velocity was 2 m s$^{-1}$ the value of $\delta a$ would be only 1/32 of the estimated value. An error of this magnitude is unlikely. For example, the rate of change in the Keplerian orbit speed at 2.8 AU for an object in a heliocentric orbit is 0.02 mm s$^{-1}$ km$^{-1}$. The AstDys estimate for the uncertainty in the semi-major axis of 2000 $EM_{61}$ is $3 \times 10^{-7}$ AU (45 km), so $\sigma_v \sim 0.001$ m s$^{-1}$. Similar error estimates may be made using the other orbital elements. Thus, the propagation errors would indicate that a highly accurate mass with an uncertainty well under 1% is likely to be obtained from observation of 2000 $EM_{61}$ with Ceres.

## 4. CONCLUSIONS

The filters developed by Hilton (2008) are quite general and can be applied to searching for encounters with any massive minor planets. It is now possible to find encounters between a massive minor planet and a perturbed minor planet that are strong enough that an accurate mass may be determined with observation periods as short as a few years.

In particular, it appears the upcoming encounter of 2000 $EM_{61}$ with Ceres will be so strong that a mass of Ceres with an accuracy of well under 1% can be determined in only a year of observation. Furthermore, the fact that such a strong encounter was found within such a short span of time indicates that it should be possible to find encounters with massive minor planets with mean diameters in the range of 150-200 km that will yield masses with $\sim 5-10\%$ uncertainties. Such minor planets individually have only about 1% the mass of Ceres, but they are much more numerous and significant perturbers of the inner planets. A search for such encounters will be the subject of further work.

## 5. REFERENCES


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