Graphical Portraits of Selected Coordinate Systems

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Contents

This document contains simple 2D and 3D plots representing certain selected coordinate systems. In each 3D plot are three surfaces, each a constant over one of the coordinates. The 2D plots are two sets of coordinate curves for a given value of the third coordinate – that is, an intersection of a specific coordinate surface with families of the other two coordinate surfaces. The coordinate systems are a small subset of ones known by Maple; they were chosen for my own peculiar reasons and are therefore not representative of anything in particular. The Maple coordinate systems are listed in the help for coords.

Click on a link to go directly to that section. Alternately, use the bookmarks for navigation (under the View menu item).

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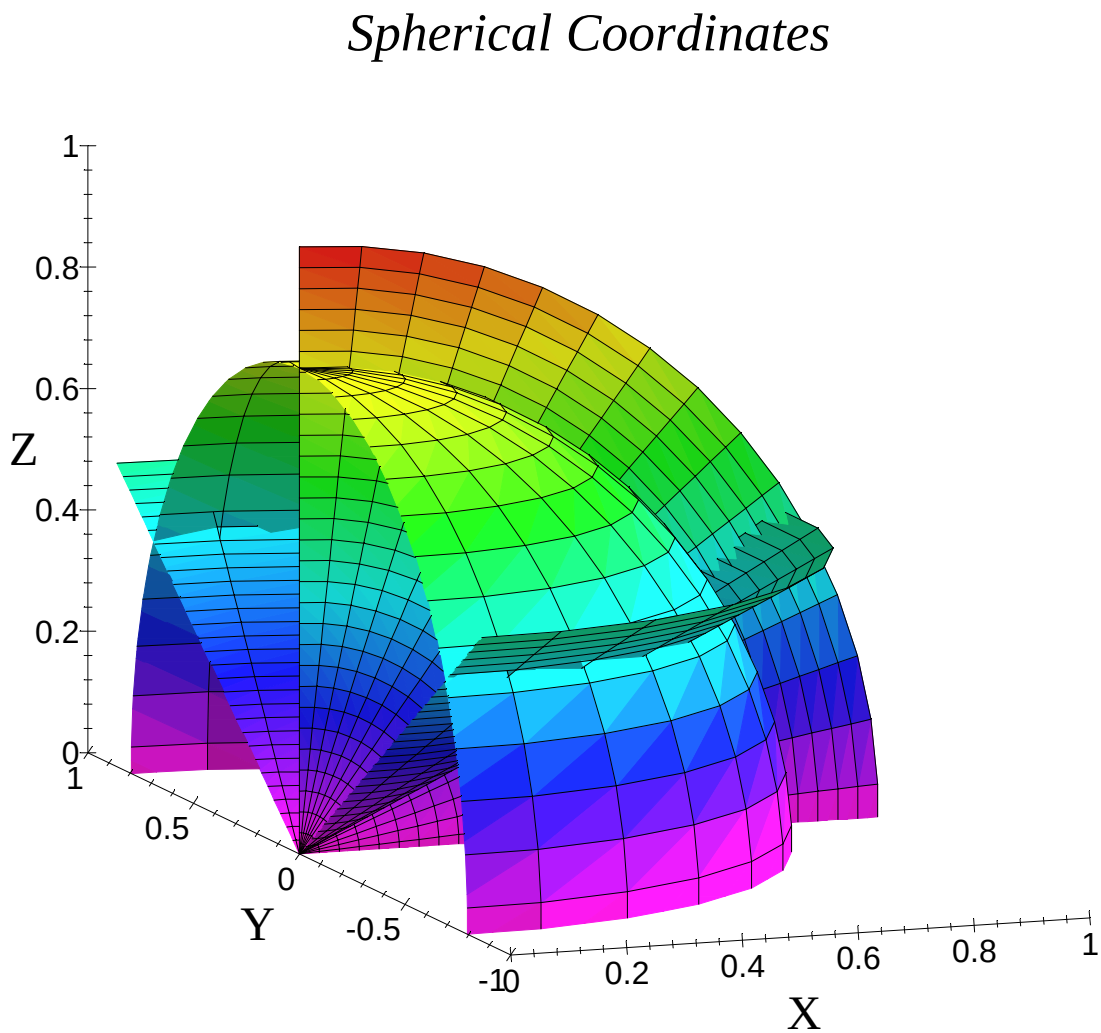
Spherical

```
spherical:
  x = u*cos(v)*sin(w)
  y = u*sin(v)*sin(w)
  z = u*cos(w)
```

```
xyz := fn( [ u*cos(v)*sin(w),
             u*sin(v)*sin(w),
             u*cos(w) ], u, v, w );
```

$$xyz := (u, v, w) \rightarrow [u \cos(v) \sin(w), u \sin(v) \sin(w), u \cos(w)]$$

```
coordplot3D( xyz, 0.8, 0, Pi/3, 0..1, -Pi..Pi, 0..Pi,
             [-110,80], [0..1, -1..1, 0..1], `Spherical Coordinates` );
```

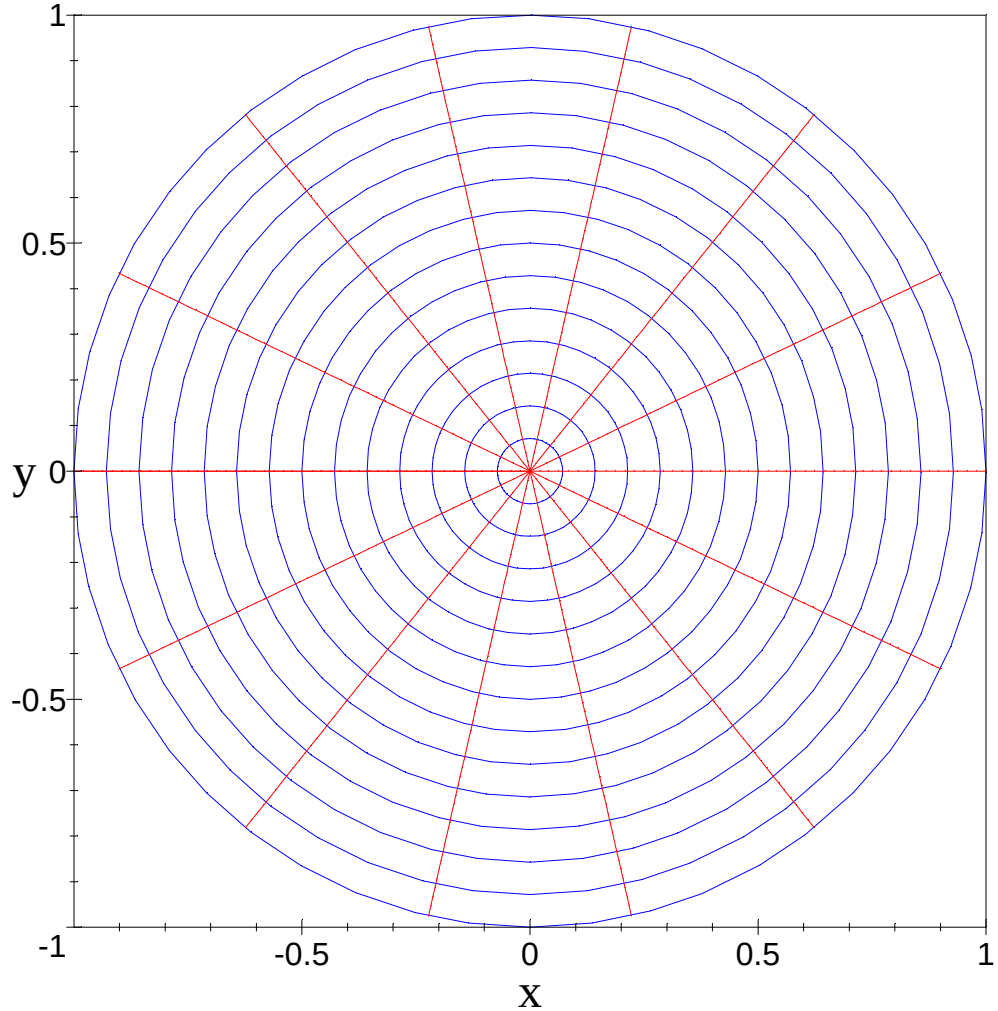



```
xy := fn( [u*cos(v),u*sin(v)], u, v );
```

$$xy := (u, v) \rightarrow [u \cos(v), u \sin(v)]$$

```
coordplot2D( xy, 0..1, -Pi..Pi,
             [-1..1,-1..1], 15,
            `Spherical Coordinates (w=pi/2)` );
```

Spherical Coordinates (w=pi/2)



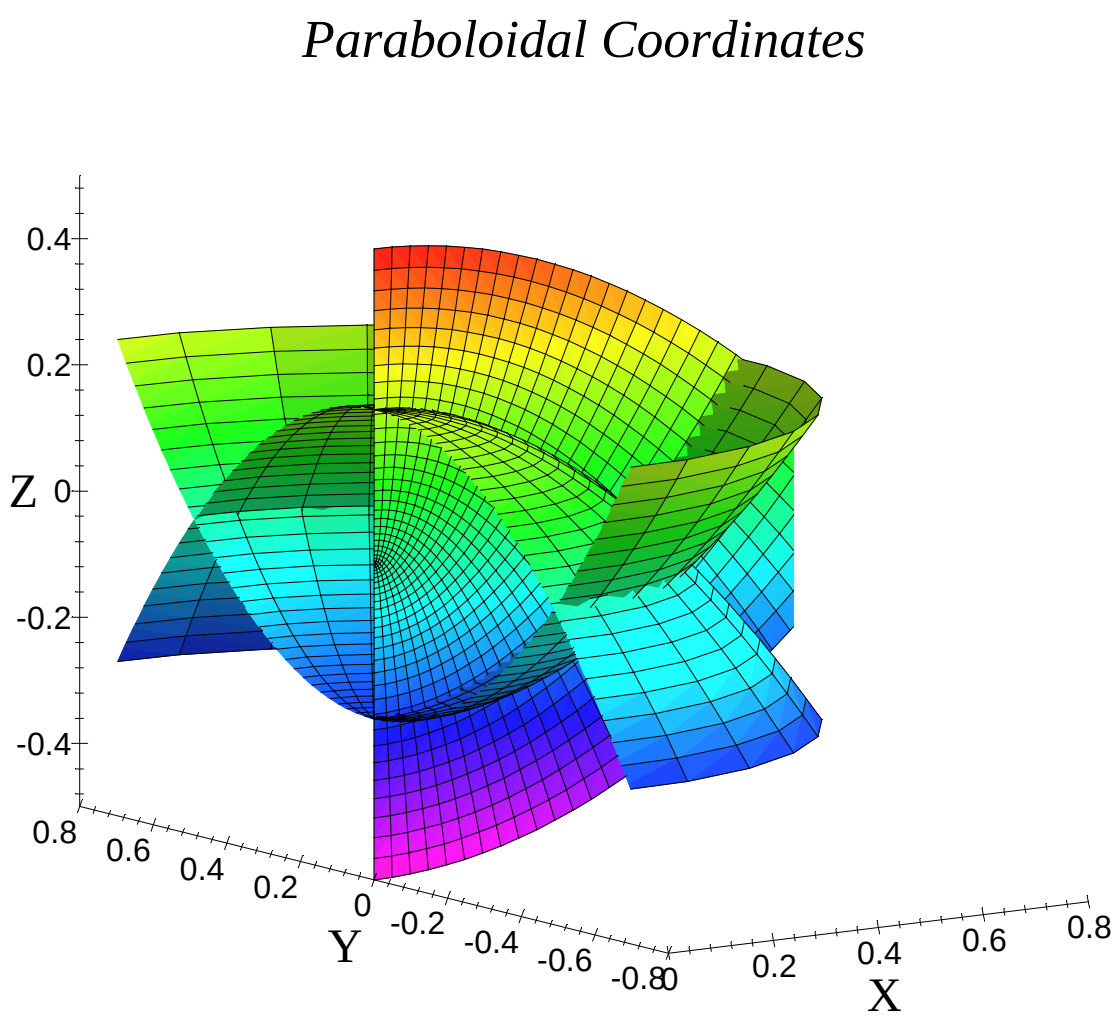
Paraboloidal

```
paraboloidal:                    ( Spiegel )
  x = u*v*cos(w)
  y = u*v*sin(w)
  z = (u^2 - v^2)/2
```

```
xyz := fn( [ u*v*cos(w),
             u*v*sin(w),
            (u^2 - v^2)/2 ], u, v, w );
```

$$xyz := (u, v, w) \rightarrow \left[u\,v\cos(w), u\,v\sin(w), \frac{1}{2}u^2 - \frac{1}{2}v^2 \right]$$

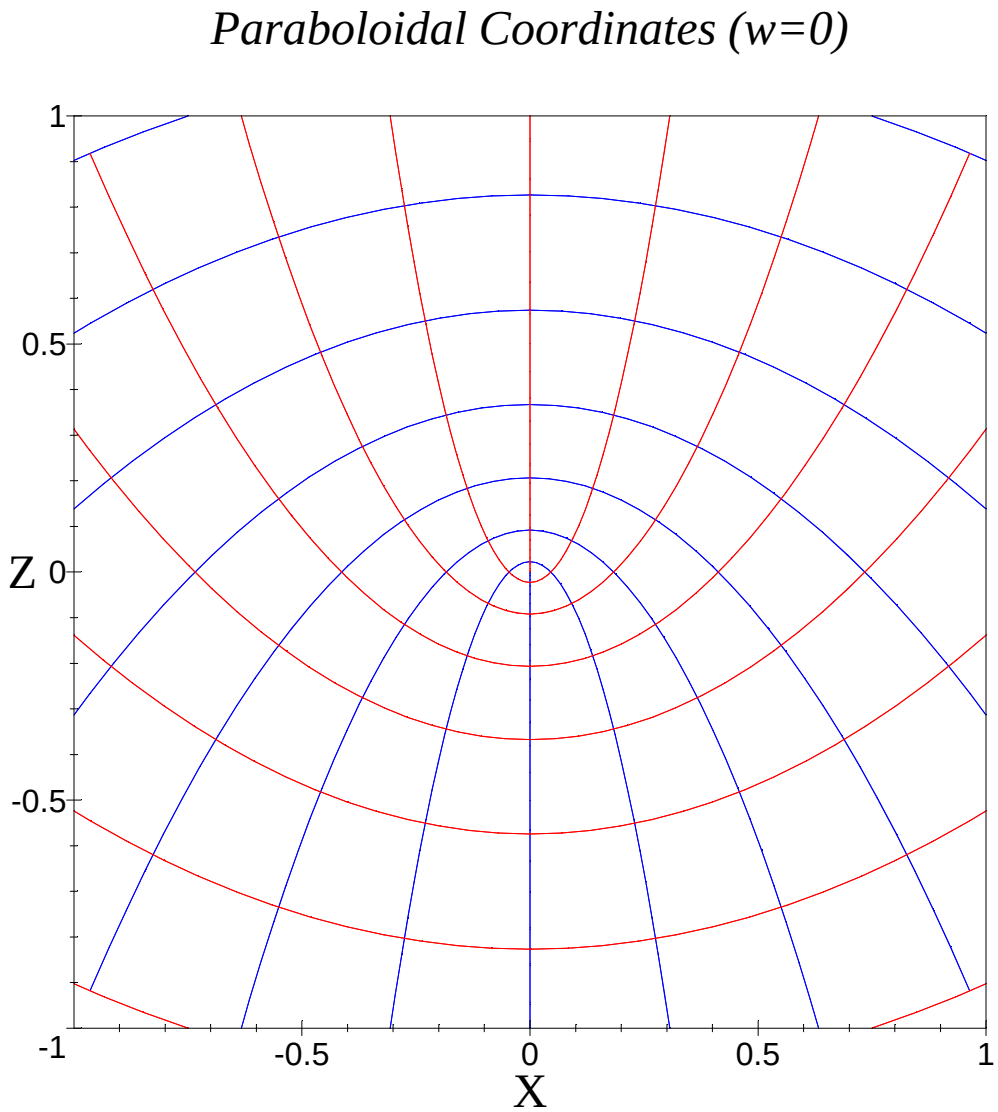
```
coordplot3D( xyz, 0.7, 0.7, 0, 0..1, 0..1, -Pi..Pi,
             [-125,80], [0..0.8, -0.8..0.8, -0.5..0.5],
            `Paraboloidal Coordinates` );
```




```
xz := fn( [u*v,1/2*u^2-1/2*v^2], u, v );
```

$$xz := (u, v) \rightarrow \left[u\,v, \frac{1}{2}u^2 - \frac{1}{2}v^2 \right]$$

```
coordplot2D( xz, -1.5..1.5, -1.5..1.5,
             [-1..1,-1..1], 15,
            `Paraboloidal Coordinates (w=0)`,
             labels=[`X`,`Z`] );
```



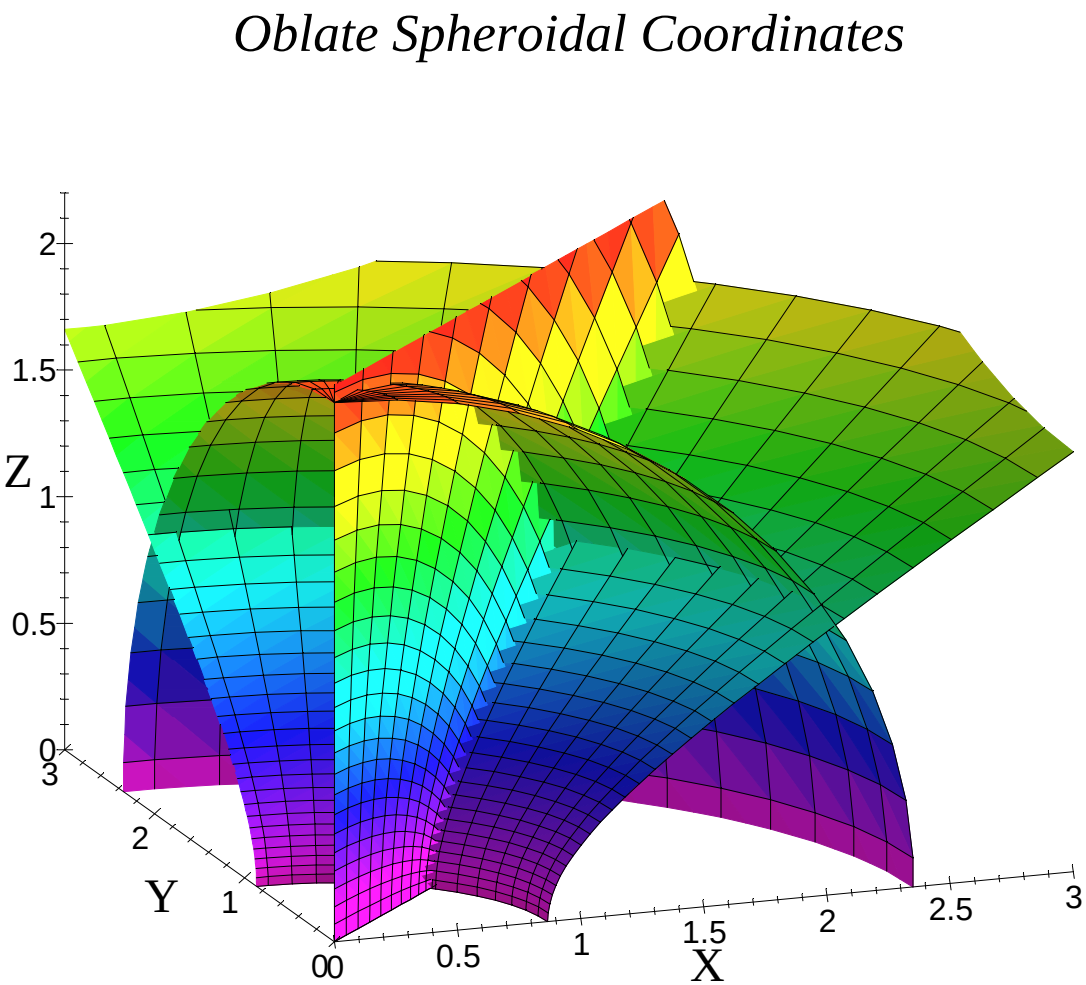
Oblate Spheroidal

```
oblatespheroidal:
 x = a*cosh(u)*sin(v)*cos(w)
 y = a*cosh(u)*sin(v)*sin(w)
 z = a*sinh(u)*cos(v)
```

```
xyz := fn( [ cosh(u)*sin(v)*cos(w),
             cosh(u)*sin(v)*sin(w),
             sinh(u)*cos(v) ],
                  u, v, w );
```

$$xyz := (u, v, w) \rightarrow [\cosh(u)\sin(v)\cos(w), \cosh(u)\sin(v)\sin(w), \sinh(u)\cos(v)]$$

```
coordplot3D( xyz, 1.5, Pi/3, Pi/4, 0..2, 0..Pi, 0..Pi,
             [-110,75], [0..3, 0..3, 0..2.2],
            `Oblate Spheroidal Coordinates` );
```



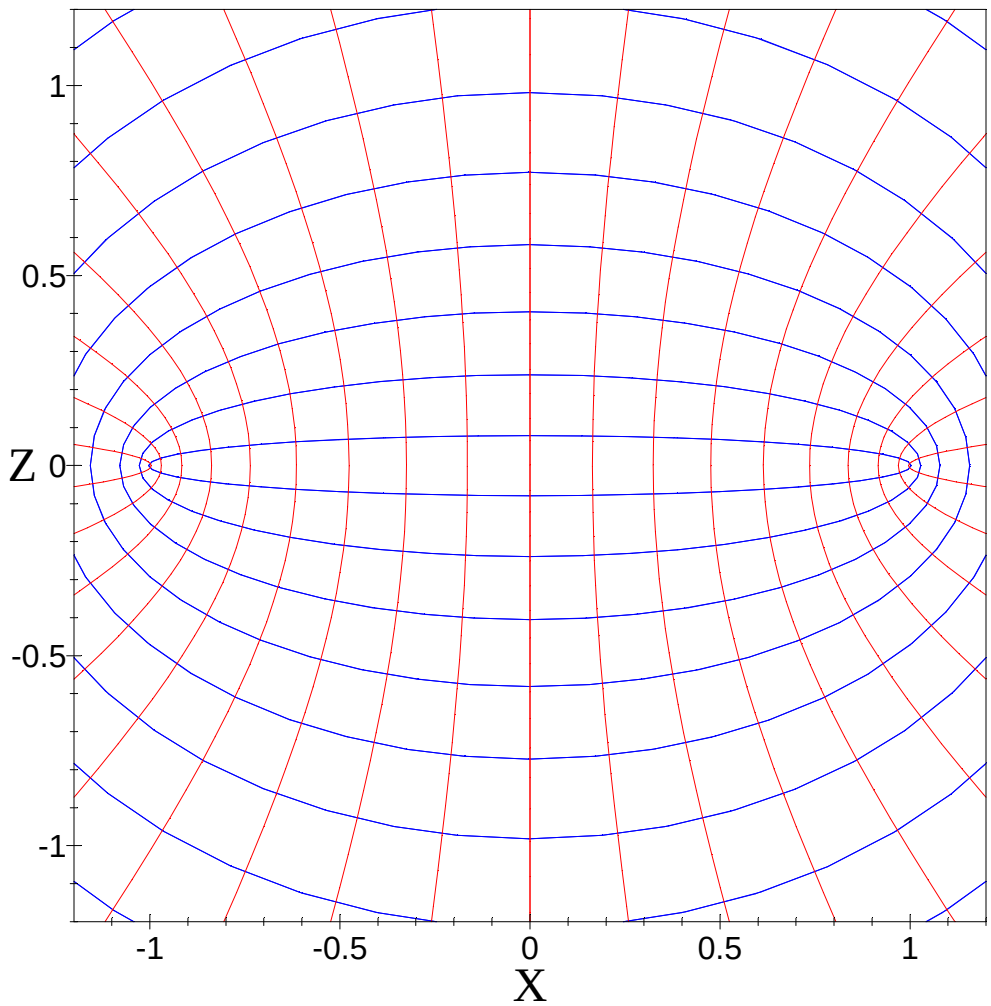

```
xz := fn( [cosh(u)*sin(v),sinh(u)*cos(v)], u, v );
```

$$xz := (u, v) \rightarrow [\cosh(u)\sin(v), \sinh(u)\cos(v)]$$

```
coordplot2D( xz, -1.5..1.5, -Pi..Pi,
             [-1.2..1.2,-1.2..1.2], 20,
```

```
`Oblate Spheroidal Coordinates (w=0)`,
 labels=[`X`,`Z`] );
```

Oblate Spheroidal Coordinates (w=0)



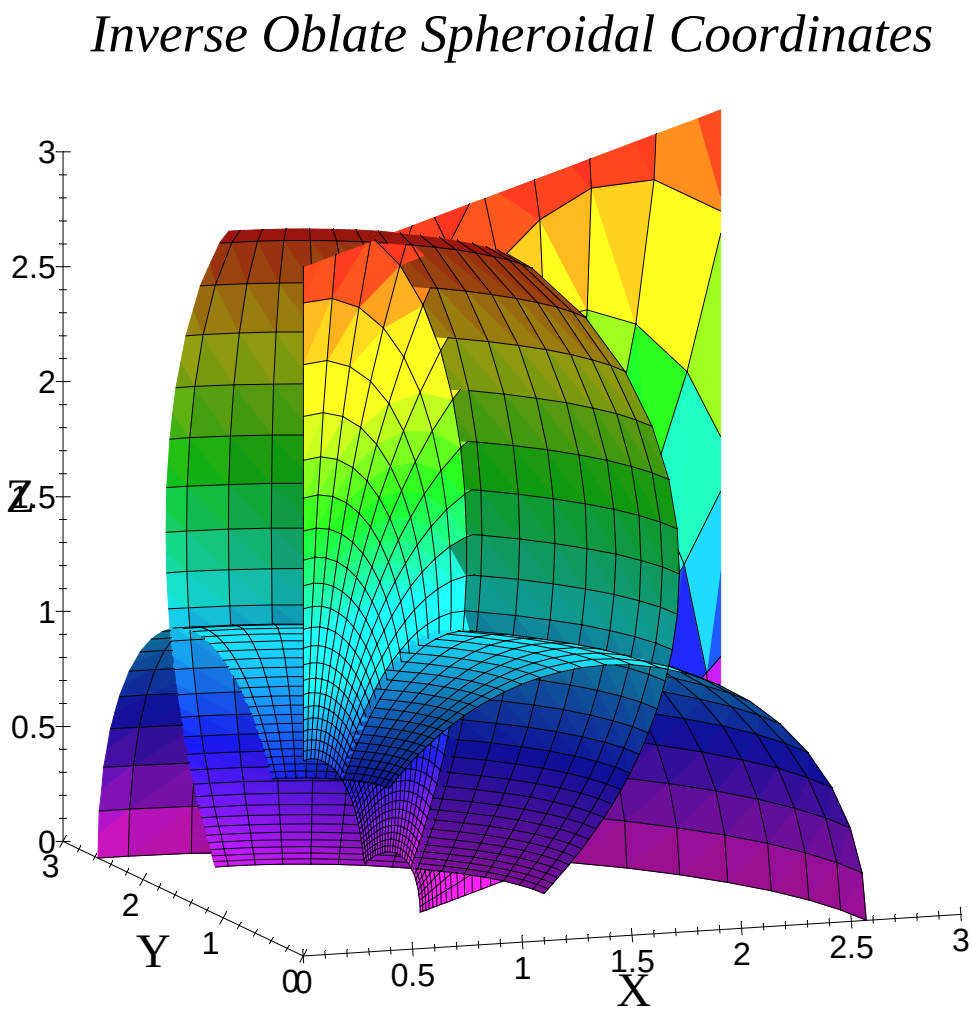
Inverse Oblate Spheroidal

```
invoblspheroidal:            (inverse oblate spheroidal)
  x = a*cosh(u)*sin(v)*cos(w)/(cosh(u)^2-cos(v)^2)
  y = a*cosh(u)*sin(v)*sin(w)/(cosh(u)^2-cos(v)^2)
  z = a*sinh(u)*cos(v)/(cosh(u)^2-cos(v)^2)
```

```
xyz := fn( [ cosh(u)*sin(v)*cos(w)/(cosh(u)^2-cos(v)^2),
             cosh(u)*sin(v)*sin(w)/(cosh(u)^2-cos(v)^2),
             sinh(u)*cos(v)/(cosh(u)^2-cos(v)^2) ],
             u, v, w );
```

$$xyz := (u, v, w) \rightarrow \left[\frac{\cosh(u)\sin(v)\cos(w)}{\cosh(u)^2 - \cos(v)^2}, \frac{\cosh(u)\sin(v)\sin(w)}{\cosh(u)^2 - \cos(v)^2}, \frac{\sinh(u)\cos(v)}{\cosh(u)^2 - \cos(v)^2} \right]$$

```
coordplot3D( xyz, 0.3, 0.4, Pi/4, 0..1, 0..1, 0..Pi,
             [-110,80], [0..3, 0..3, 0..3],
            `Inverse Oblate Spheroidal Coordinates` );
```

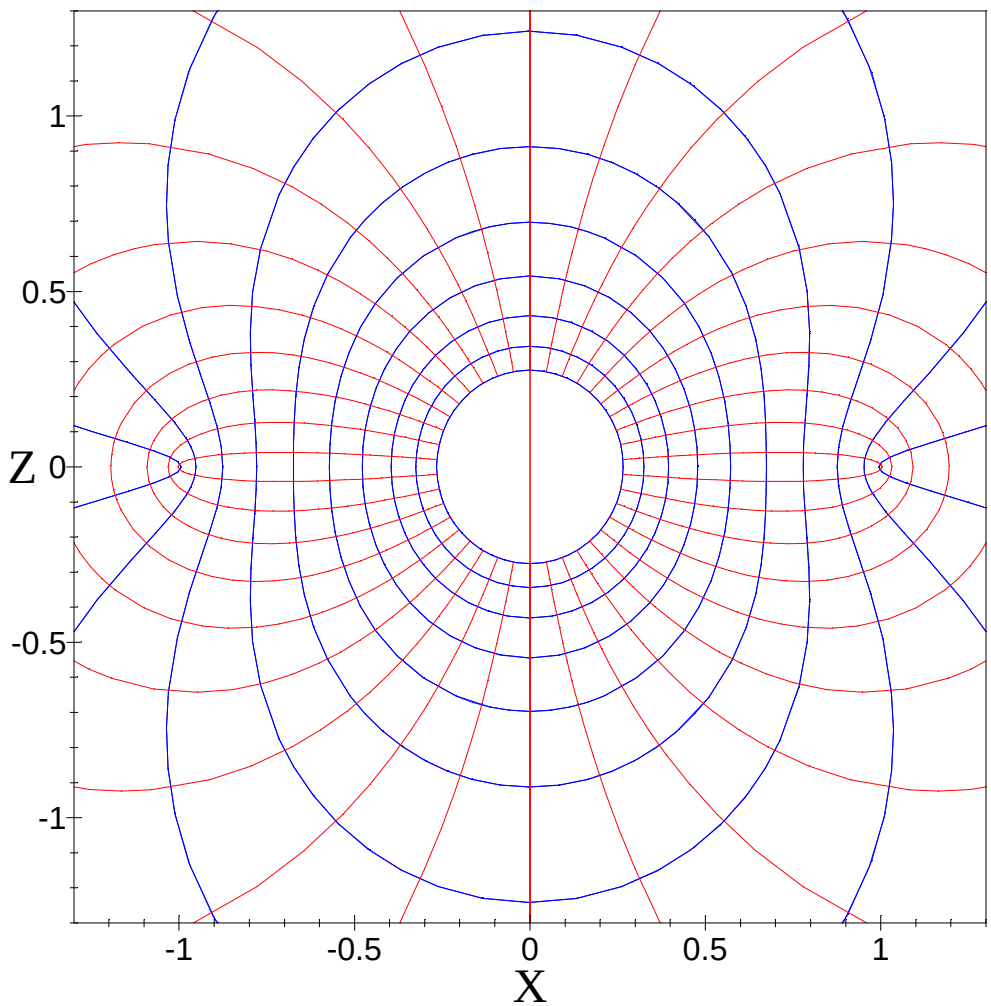



```
xz := fn( [cosh(u)*sin(v)/(cosh(u)^2-cos(v)^2),
           sinh(u)*cos(v)/(cosh(u)^2-cos(v)^2)], u, v );
```

$$xz := (u, v) \rightarrow \left[\frac{\cosh(u)\sin(v)}{\cosh(u)^2 - \cos(v)^2}, \frac{\sinh(u)\cos(v)}{\cosh(u)^2 - \cos(v)^2} \right]$$

```
coordplot2D( xz, -2..2, -Pi..Pi,
             [-1.3..1.3,-1.3..1.3], 20,
            `Inverse Oblate Spheroidal Coordinates (w=0)`,
             labels=[`X`,`Z`] );
```

Inverse Oblate Spheroidal Coordinates (w=0)



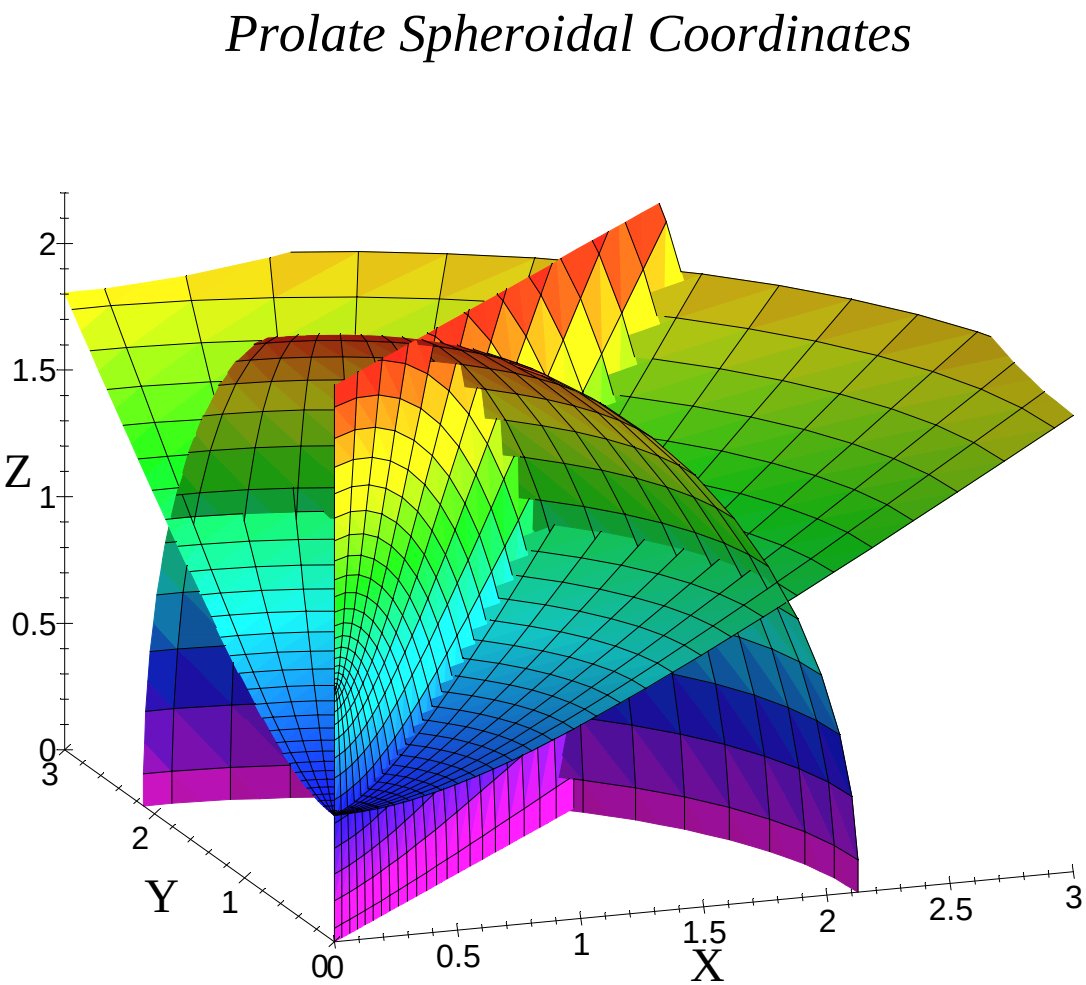
Prolate Spheroidal

```
prolatespheroidal:
  x = a*sinh(u)*sin(v)*cos(w)
  y = a*sinh(u)*sin(v)*sin(w)
  z = a*cosh(u)*cos(v)
```

```
xyz := fn( [ sinh(u)*sin(v)*cos(w),
             sinh(u)*sin(v)*sin(w),
             cosh(u)*cos(v) ],
             u, v, w );
```

$$xyz := (u, v, w) \rightarrow [\sinh(u)\sin(v)\cos(w), \sinh(u)\sin(v)\sin(w), \cosh(u)\cos(v)]$$

```
coordplot3D( xyz, 1.5, Pi/3, Pi/4, 0..2, 0..Pi, 0..Pi,
             [-110,75], [0..3, 0..3, 0..2.2],
            `Prolate Spheroidal Coordinates` );
```



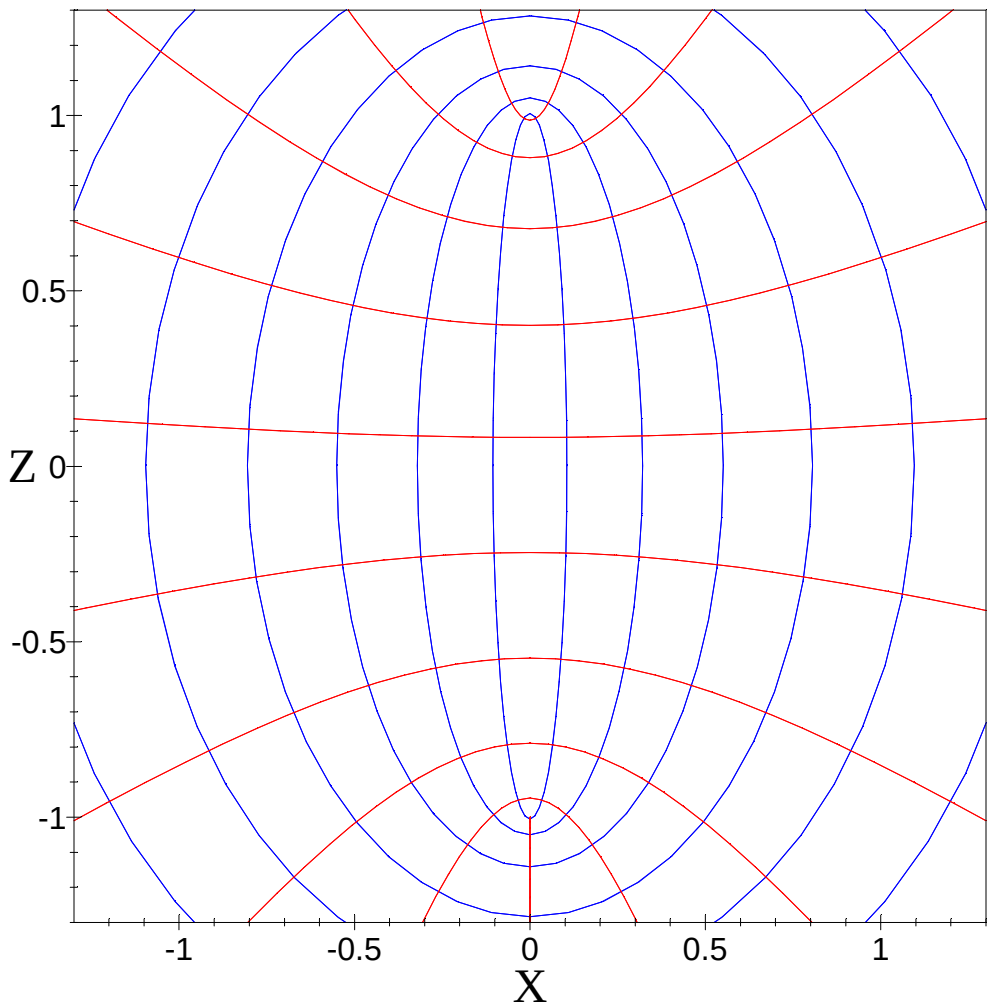

```
xz := fn( [sinh(u)*sin(v),
           cosh(u)*cos(v)], u, v );
```

$$xz := (u, v) \rightarrow [\sinh(u)\sin(v), \cosh(u)\cos(v)]$$

```
coordplot2D( xz, -2..2, -Pi..Pi,
```

```
 [-1.3..1.3,-1.3..1.3], 20,
`Prolate Spheroidal Coordinates (w=0)`,
 labels=[`X`,`Z`] );
```

Prolate Spheroidal Coordinates (w=0)



Inverse Prolate Spheroidal

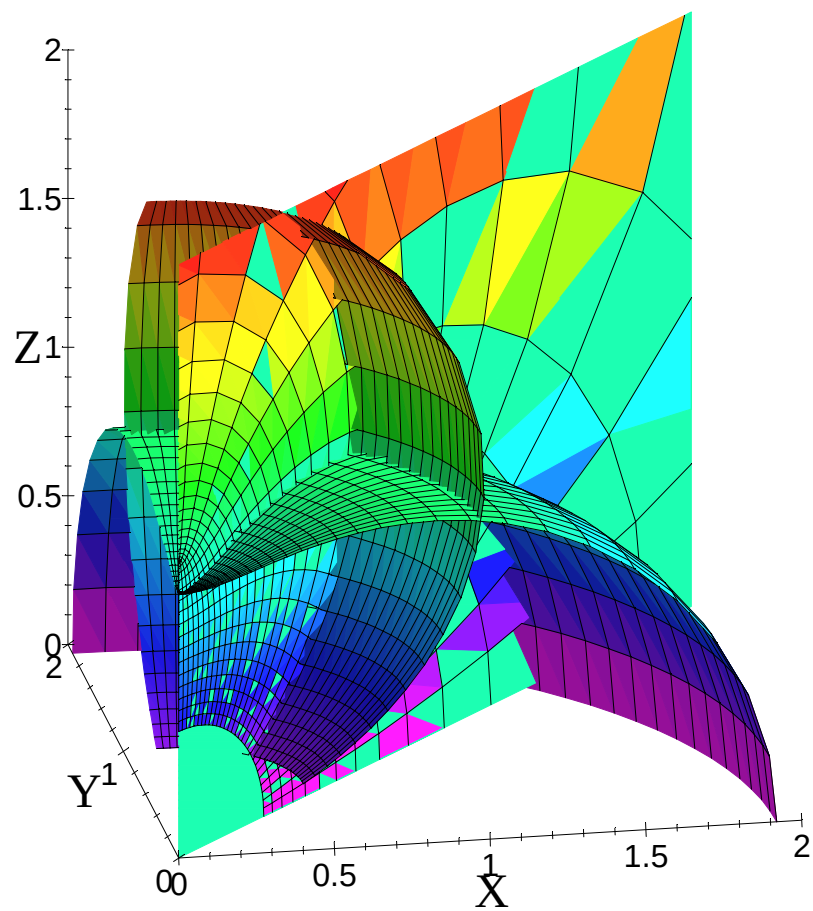
```
invprospheroidal:            (inverse prolate spheroidal)
  x = a*sinh(u)*sin(v)*cos(w)/(cosh(u)^2-sin(v)^2)
  y = a*sinh(u)*sin(v)*sin(w)/(cosh(u)^2-sin(v)^2)
  z = a*cosh(u)*cos(v)/(cosh(u)^2-sin(v)^2)
```

```
xyz := fn( [ sinh(u)*sin(v)*cos(w)/(cosh(u)^2-sin(v)^2),
             sinh(u)*sin(v)*sin(w)/(cosh(u)^2-sin(v)^2),
             cosh(u)*cos(v)/(cosh(u)^2-sin(v)^2) ],
             u, v, w );
```

$$xyz := (u, v, w) \rightarrow \left[\frac{\sinh(u)\sin(v)\cos(w)}{\cosh(u)^2 - \sin(v)^2}, \frac{\sinh(u)\sin(v)\sin(w)}{\cosh(u)^2 - \sin(v)^2}, \frac{\cosh(u)\cos(v)}{\cosh(u)^2 - \sin(v)^2} \right]$$

```
coordplot3D( xyz, 0.5, 1.1, Pi/4, -1..1.5, 0..Pi/2, 0..Pi/2,
             [-100,70], [0..2, 0..2, 0..2],
            `Inverse Prolate Spheroidal Coordinates` );
```

Inverse Prolate Spheroidal Coordinates

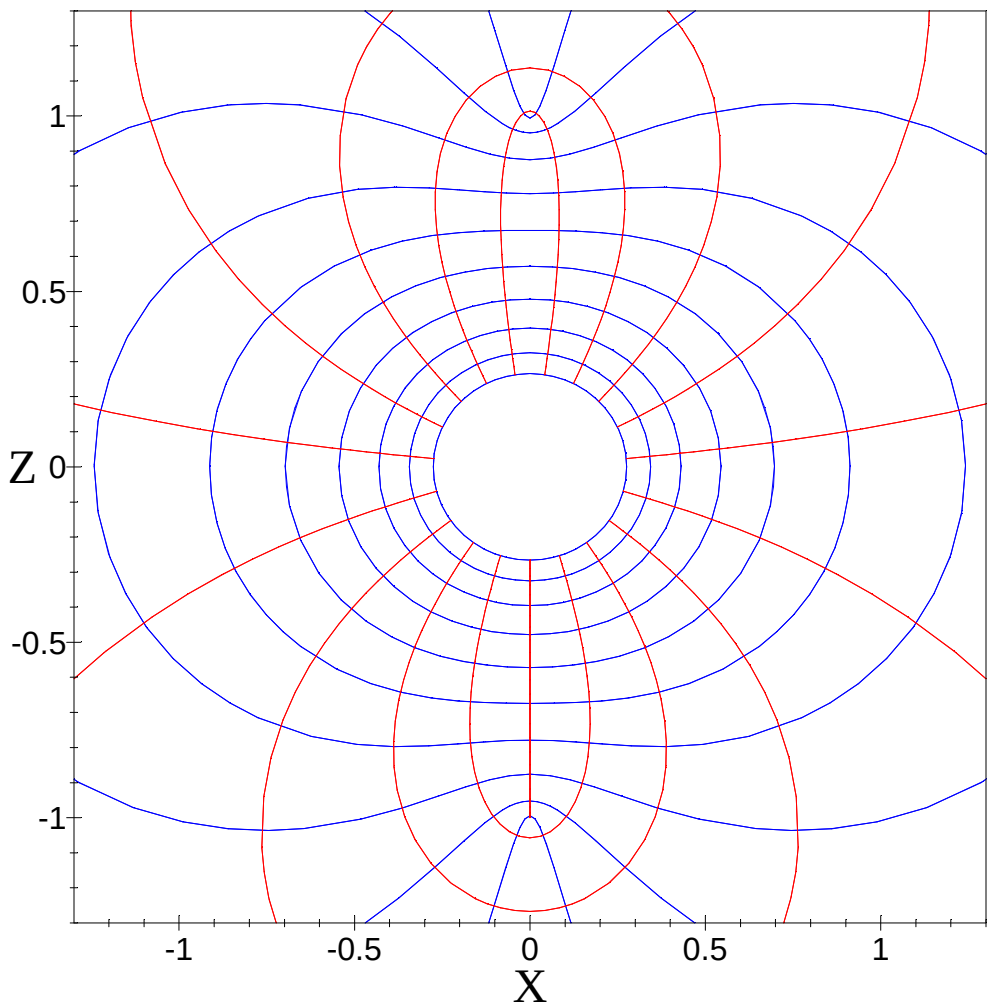



```
xz := fn( [sinh(u)*sin(v)/(cosh(u)^2-sin(v)^2),
           cosh(u)*cos(v)/(cosh(u)^2-sin(v)^2)], u, v );
```

$$xz := (u, v) \rightarrow \left[\frac{\sinh(u)\sin(v)}{\cosh(u)^2 - \sin(v)^2}, \frac{\cosh(u)\cos(v)}{\cosh(u)^2 - \sin(v)^2} \right]$$

```
coordplot2D( xz, -2..2, -Pi..Pi,
             [-1.3..1.3,-1.3..1.3], 20,
            `Inverse Prolate Spheroidal Coordinates (w=0)`,
             labels=[`X`,`Z`] );
```

Inverse Prolate Spheroidal Coordinates (w=0)



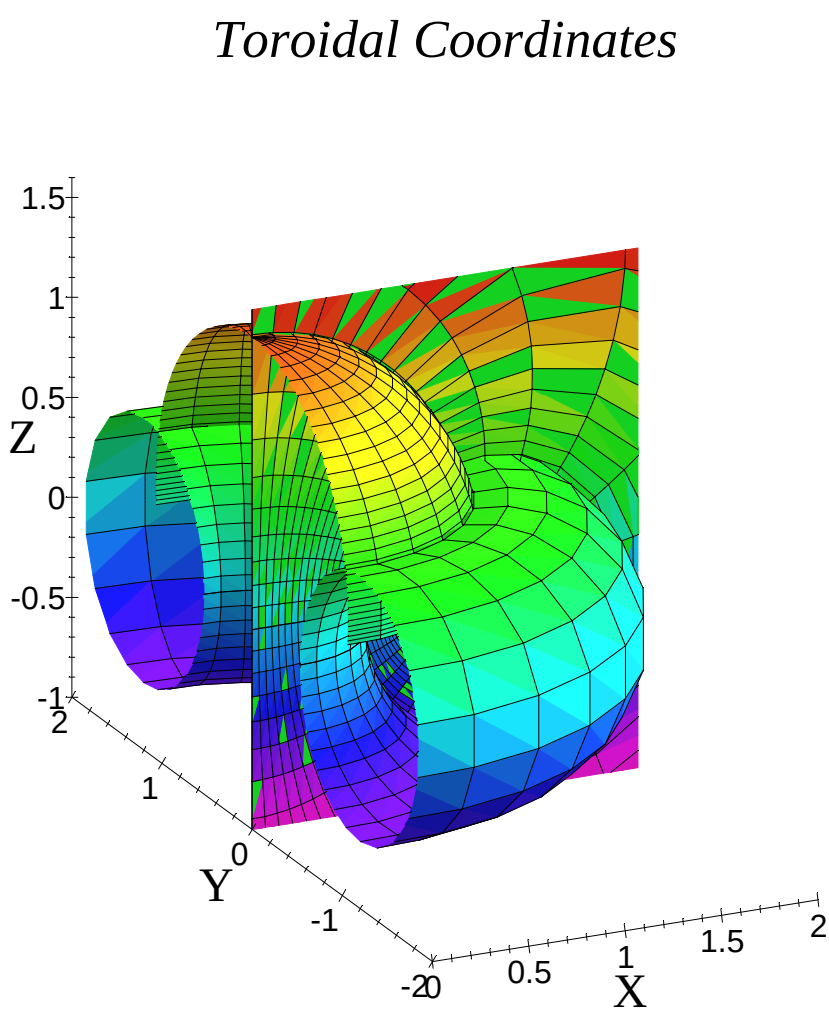
Toroidal

```
toroidal:
  x = a*sinh(v)*cos(w)/d
  y = a*sinh(v)*sin(w)/d
  z = a*sin(u)/d              ( where d = cosh(v) - cos(u) )
```

```
xyz := fn( [ sinh(v)*cos(w)/(cosh(v)-cos(u)),
             sinh(v)*sin(w)/(cosh(v)-cos(u)),
             sin(u)/(cosh(v)-cos(u)) ], u, v, w );
```

$$xyz := (u, v, w) \rightarrow \left[\frac{\sinh(v)\cos(w)}{\cosh(v) - \cos(u)}, \frac{\sinh(v)\sin(w)}{\cosh(v) - \cos(u)}, \frac{\sin(u)}{\cosh(v) - \cos(u)} \right]$$

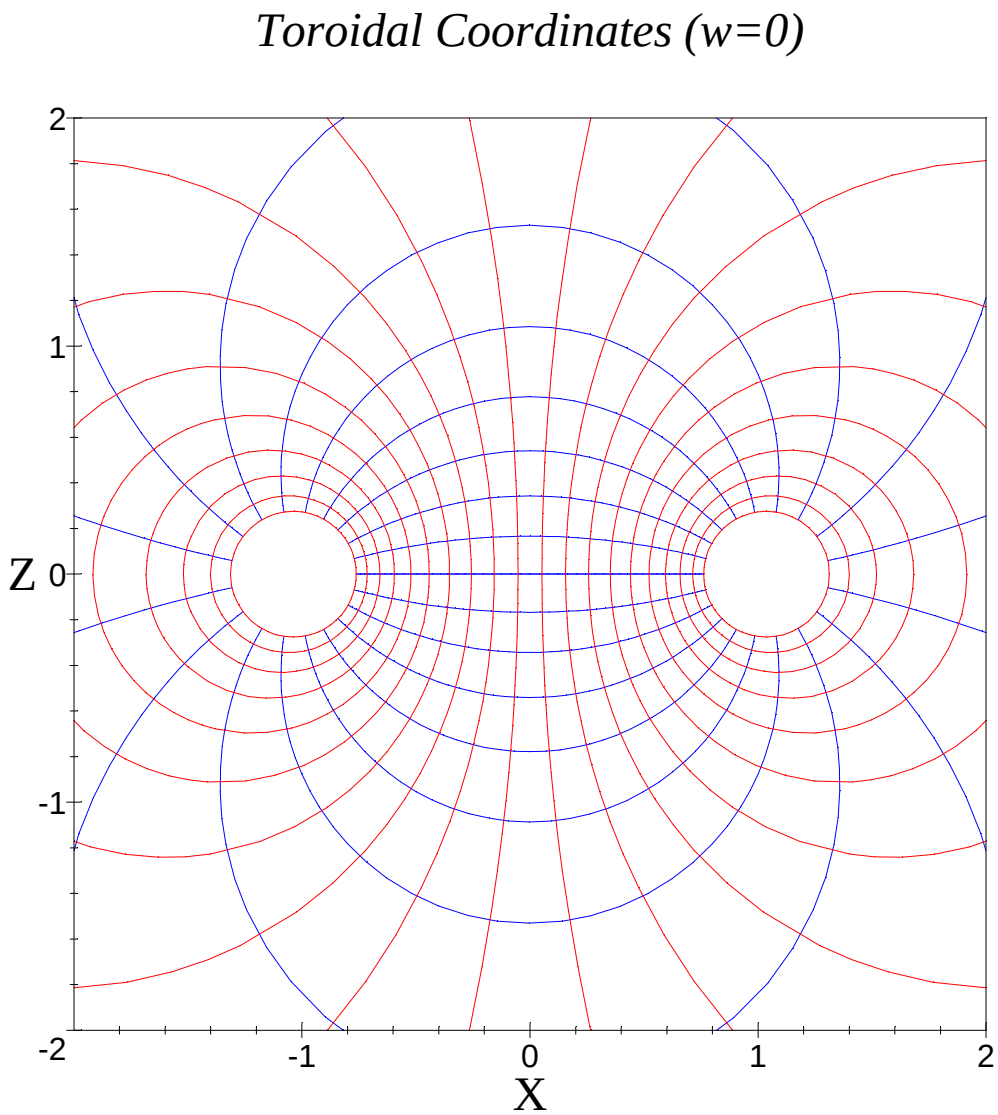
```
coordplot3D( xyz, 1.2, 1.2, 0, -Pi..Pi, 0..2, -Pi..Pi,
             [-115,70], [0..2, -2..2, -1..1.6],
            `Toroidal Coordinates` );
```




```
xz := fn( [sinh(v)/(cosh(v)-cos(u)),
           sin(u)/(cosh(v)-cos(u))], u, v );
```

$$xz := (u, v) \rightarrow \left[\frac{\sinh(v)}{\cosh(v) - \cos(u)}, \frac{\sin(u)}{\cosh(v) - \cos(u)} \right]$$

```
coordplot2D( xz, -Pi..Pi, -2..2,
             [-2..2,-2..2], 20,
            `Toroidal Coordinates (w=0)`,
             labels=[`X`,`Z`] );
```



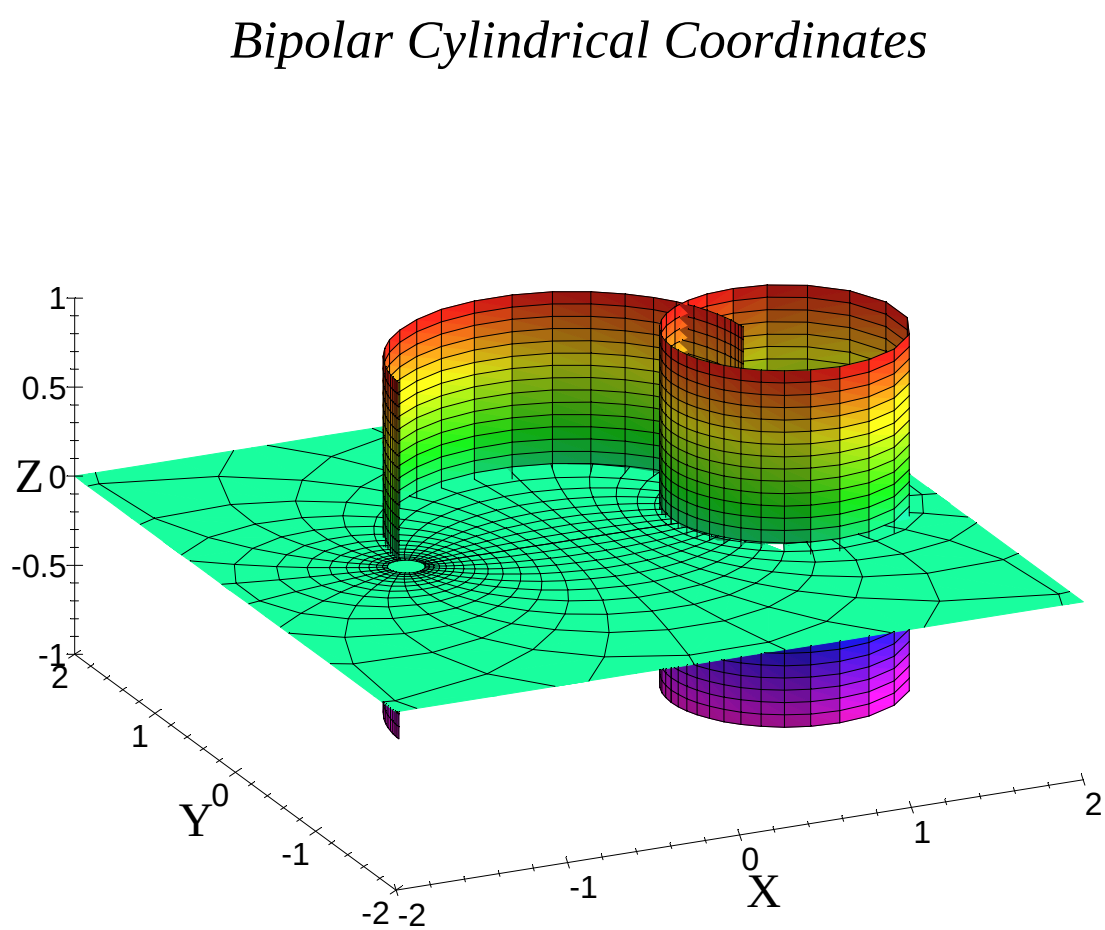
Bipolar Cylindrical

```
bipolarcylindrical:          ( Spiegel )
   x = a*sinh(v)/(cosh(v)-cos(u))
   y = a*sin(u)/(cosh(v)-cos(u))
   z = w
```

```
xyz := fn( [ sinh(v)/(cosh(v)-cos(u)),
             sin(u)/(cosh(v)-cos(u)), w ], u,v,w );
```

$$xyz := (u, v, w) \rightarrow \left[\frac{\sinh(v)}{\cosh(v) - \cos(u)}, \frac{\sin(u)}{\cosh(v) - \cos(u)}, w\right]$$

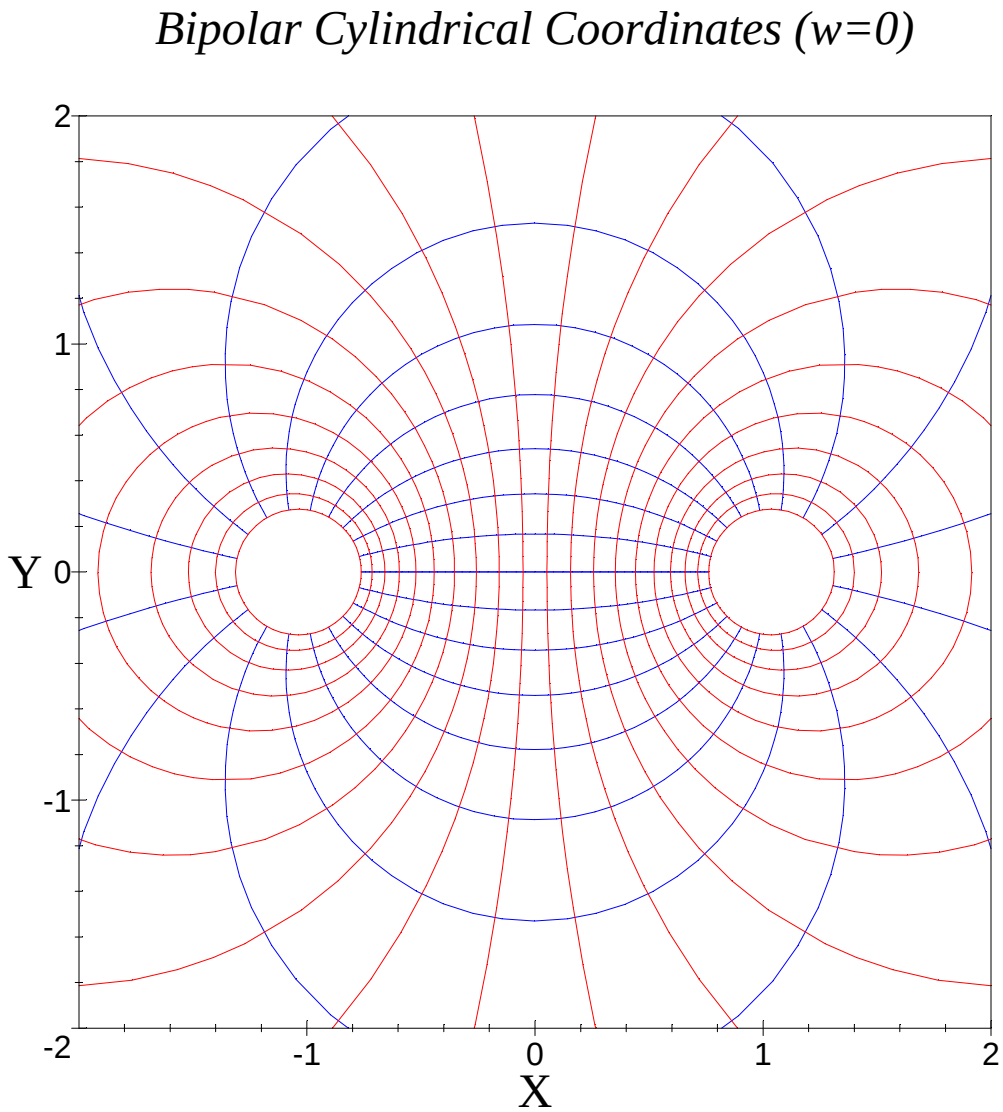
```
coordplot3D( xyz, 1.5, 1.2, 0, -Pi..Pi, -3..3, -1..1,
             [-115,70], [-2..2, -2..2, -1..1],
            `Bipolar Cylindrical Coordinates` );
```




```
xy := fn( [sinh(v)/(cosh(v)-cos(u)),
           sin(u)/(cosh(v)-cos(u))], u, v );
```

$$xy := (u, v) \rightarrow \left[\frac{\sinh(v)}{\cosh(v) - \cos(u)}, \frac{\sin(u)}{\cosh(v) - \cos(u)}\right]$$

```
coordplot2D( xy, -Pi..Pi, -2..2,
             [-2..2,-2..2], 20,
            `Bipolar Cylindrical Coordinates (w=0)`,
             labels=[`X`,`Y`] );
```



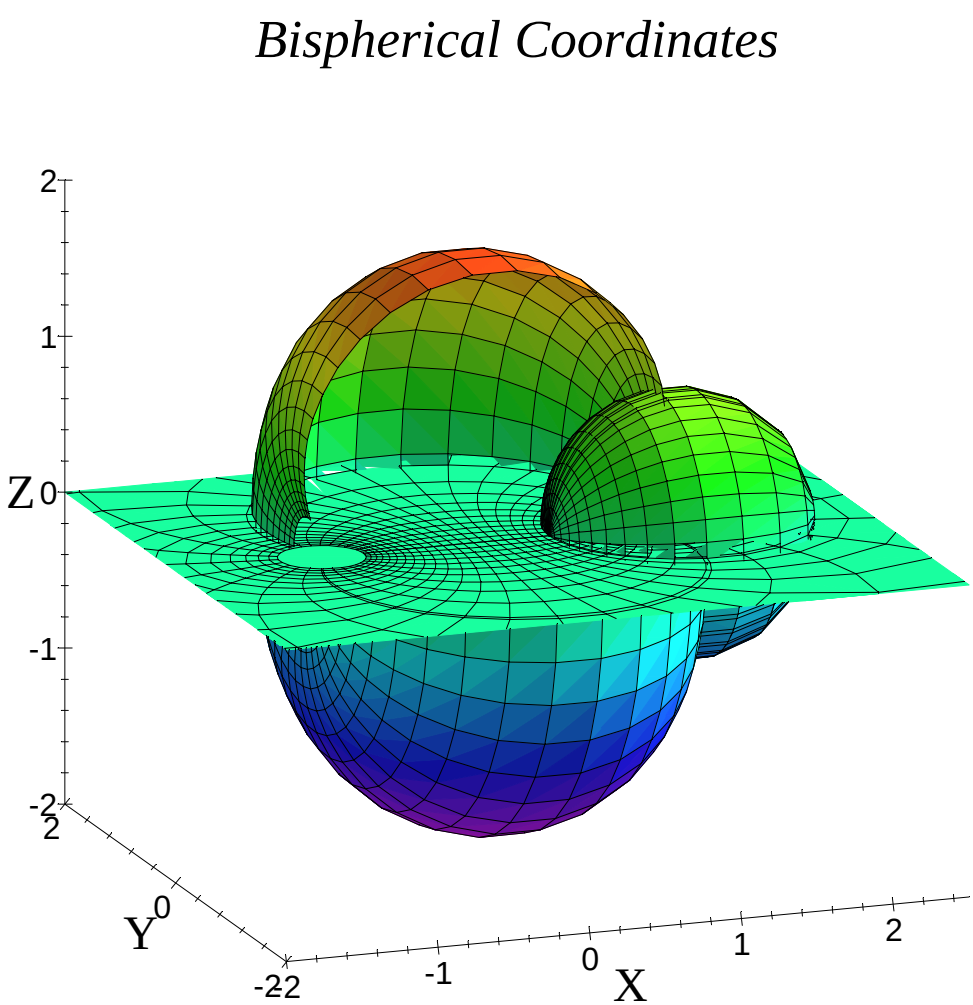
Bispherical

```
bispherical:
  x = sinh(v)/d
  y = sin(u)*cos(w)/d
  z = sin(u)*sin(w)/d              ( where d = cosh(v) - cos(u) )
```

```
xyz := fn( [ sinh(v)/(cosh(v)-cos(u)),
             sin(u)*cos(w)/(cosh(v)-cos(u)),
             sin(u)*sin(w)/(cosh(v)-cos(u)) ], u,v,w );
```

$$xyz := (u, v, w) \rightarrow \left[\frac{\sinh(v)}{\cosh(v) - \cos(u)}, \frac{\sin(u)\cos(w)}{\cosh(v) - \cos(u)}, \frac{\sin(u)\sin(w)}{\cosh(v) - \cos(u)}\right]$$

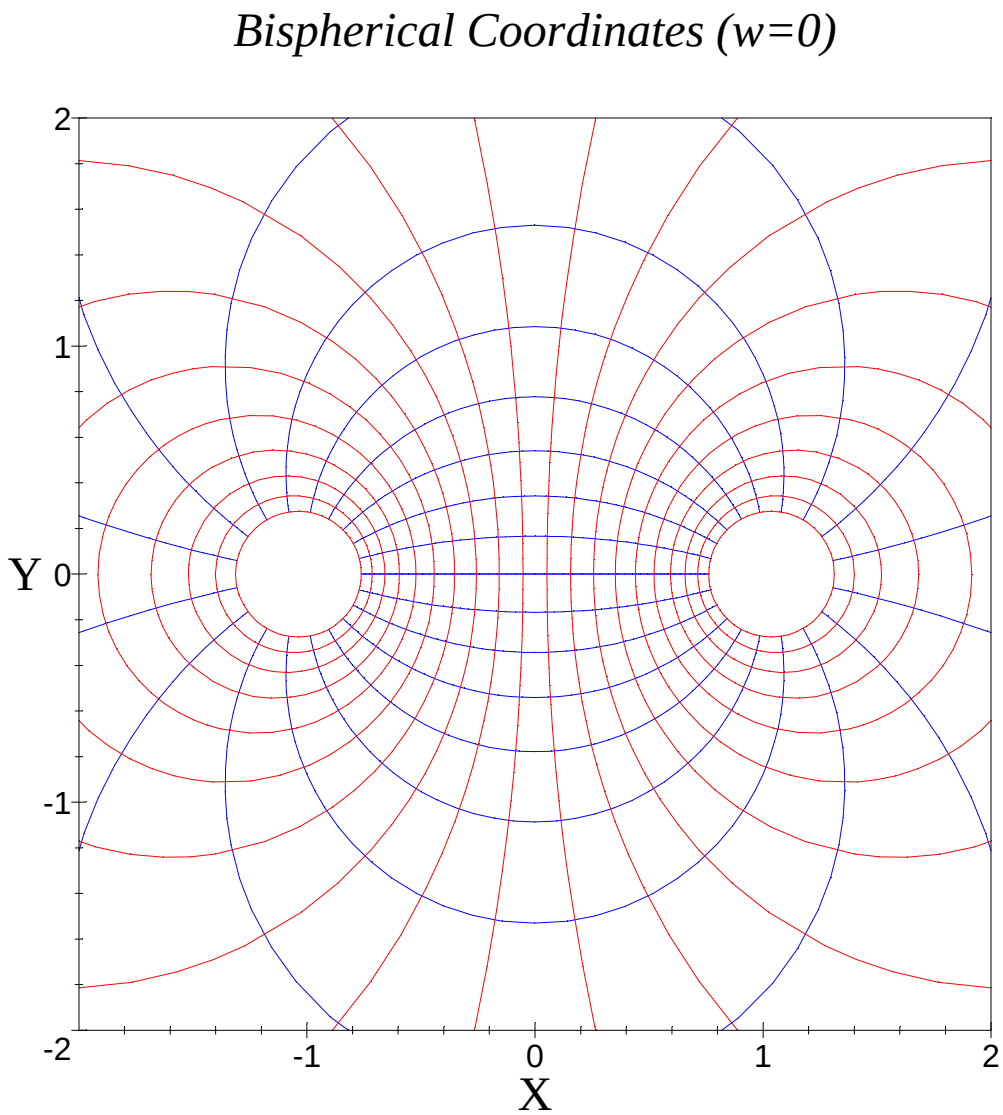
```
coordplot3D( xyz, 1, 1, 0, -Pi..Pi, -2..2, -Pi..1.1*Pi/2,
             [-110,75], [-2..2.5, -2..2, -2..2],
            `Bispherical Coordinates` );
```




```
xy := fn( [sinh(v)/(cosh(v)-cos(u)),
           sin(u)/(cosh(v)-cos(u))], u, v );
```

$$xy := (u, v) \rightarrow \left[\frac{\sinh(v)}{\cosh(v) - \cos(u)}, \frac{\sin(u)}{\cosh(v) - \cos(u)}\right]$$

```
coordplot2D( xy, -Pi..Pi, -2..2,
             [-2..2,-2..2], 20,
            `Bispherical Coordinates (w=0)`,
             labels=[`X`,`Y`] );
```



Tangent Spheroidal

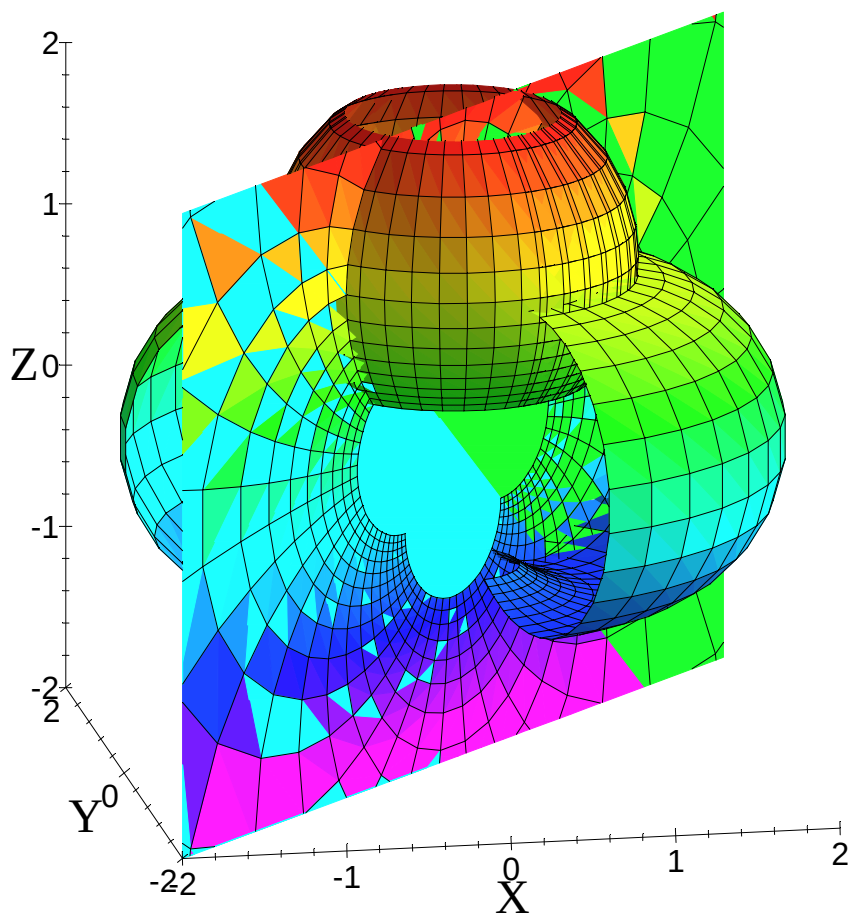
```
tangentsphere:
  x = u*cos(w)/(u^2+v^2)
  y = u*sin(w)/(u^2+v^2)
  z = v/(u^2+v^2)
```

```
xyz := fn( [ u*cos(w)/(u^2+v^2),
             u*sin(w)/(u^2+v^2),
             v/(u^2+v^2) ], u,v,w );
```

$$xyz := (u, v, w) \rightarrow \left[\frac{u\cos(w)}{u^2+v^2}, \frac{u\sin(w)}{u^2+v^2}, \frac{v}{u^2+v^2}\right]$$

```
coordplot3D( xyz, 0.5, 0.45, Pi/4, -1..1, -1..1, -0.4*Pi..Pi,
             [-100,75], [-2..2, -2..2, -2..2],
            `Tangent Spheroidal Coordinates` );
```

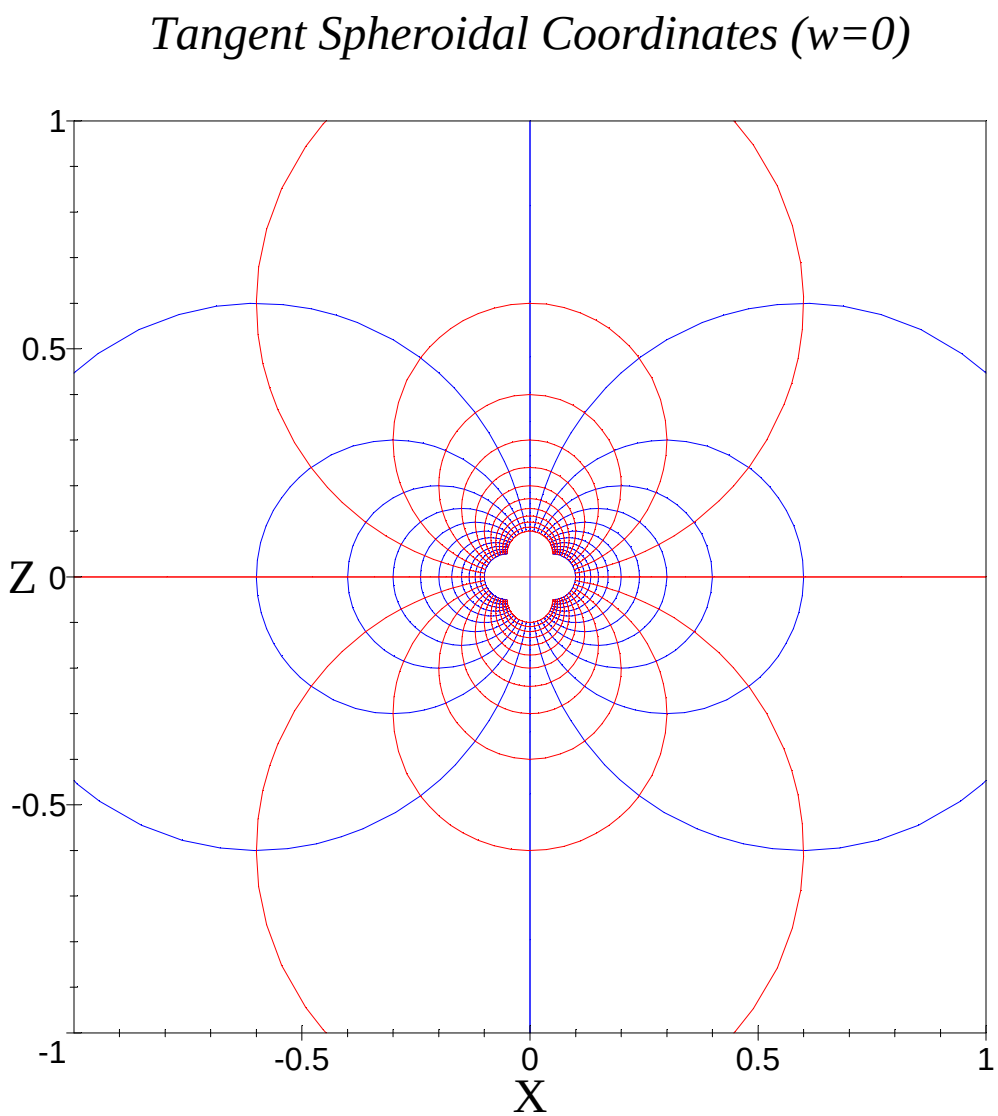
Tangent Spheroidal Coordinates




```
xz := fn( [u/(u^2+v^2), v/(u^2+v^2)], u, v );
```

$$xz := (u, v) \rightarrow \left[\frac{u}{u^2 + v^2}, \frac{v}{u^2 + v^2}\right]$$

```
coordplot2D( xz, -10..10, -10..10,
             [-1..1,-1..1], 25,
            `Tangent Spheroidal Coordinates (w=0)`,
             labels=[`X`,`Z`] );
```



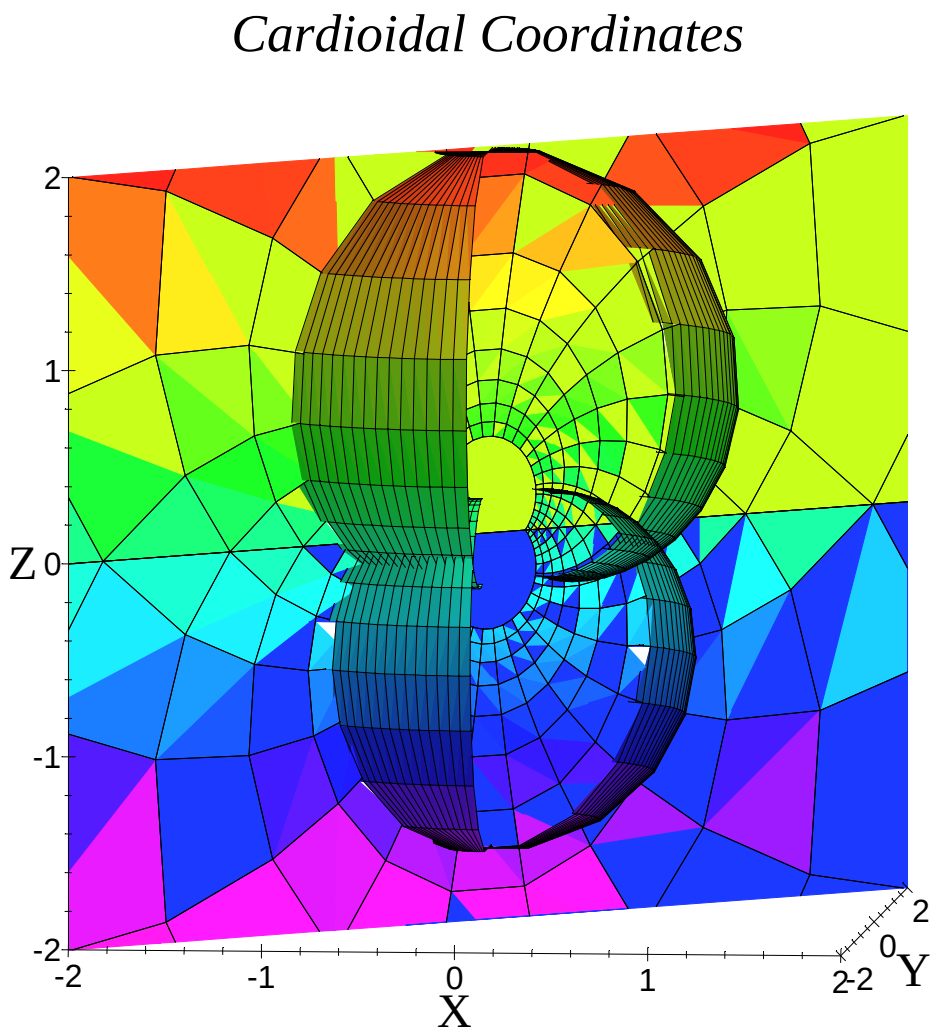
Cardioidal

```
cardiodal:
  x = u*v*cos(w)/(u^2+v^2)^2
  y = u*v*sin(w)/(u^2+v^2)^2
  z = (u^2-v^2)/2/(u^2+v^2)^2
```

```
xyz := fn( [ u*v*cos(w)/(u^2+v^2)^2,
             u*v*sin(w)/(u^2+v^2)^2,
            (u^2-v^2)/2/(u^2+v^2)^2 ], u,v,w );
```

$$xyz := (u, v, w) \rightarrow \left[\frac{u\,v\cos(w)}{(u^2+v^2)^2}, \frac{u\,v\sin(w)}{(u^2+v^2)^2}, \frac{1}{2}\frac{u^2-v^2}{(u^2+v^2)^2} \right]$$

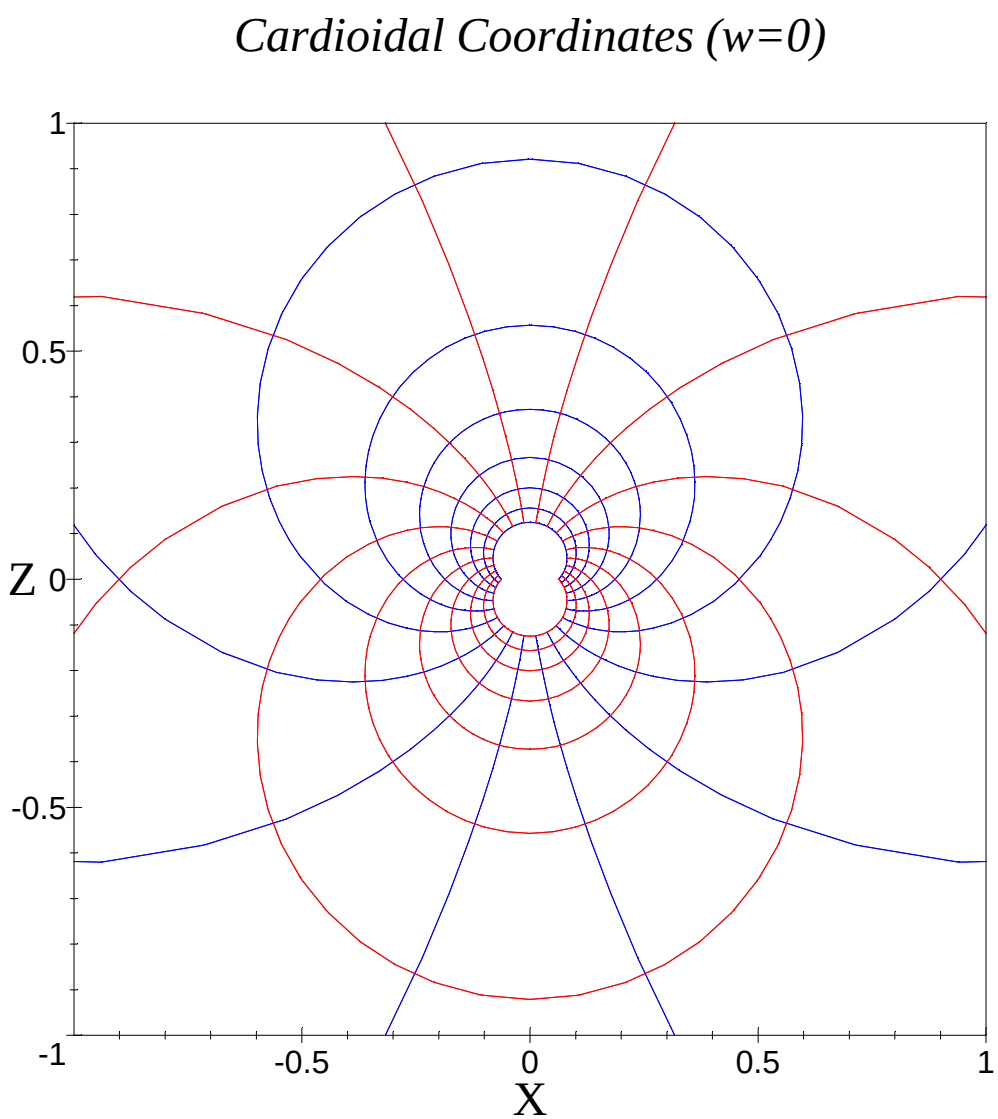
```
coordplot3D( xyz, 0.5, 0.55, Pi/4, -1..1, -1..1, 0..Pi/2,
             [-85,85], [-2..2, -2..2, -2..2],
            `Cardioidal Coordinates` );
```




```
xz := fn( [u*v/(u^2+v^2)^2,
          (u^2-v^2)/2/(u^2+v^2)^2], u, v );
```

$$xz := (u, v) \rightarrow \left[\frac{u\,v}{(u^2+v^2)^2}, \frac{1}{2}\frac{u^2-v^2}{(u^2+v^2)^2} \right]$$

```
coordplot2D( xz, -2..2, -2..2,
             [-1..1,-1..1], 20,
            `Cardioidal Coordinates (w=0)`,
             labels=[`X`,`Z`] );
```



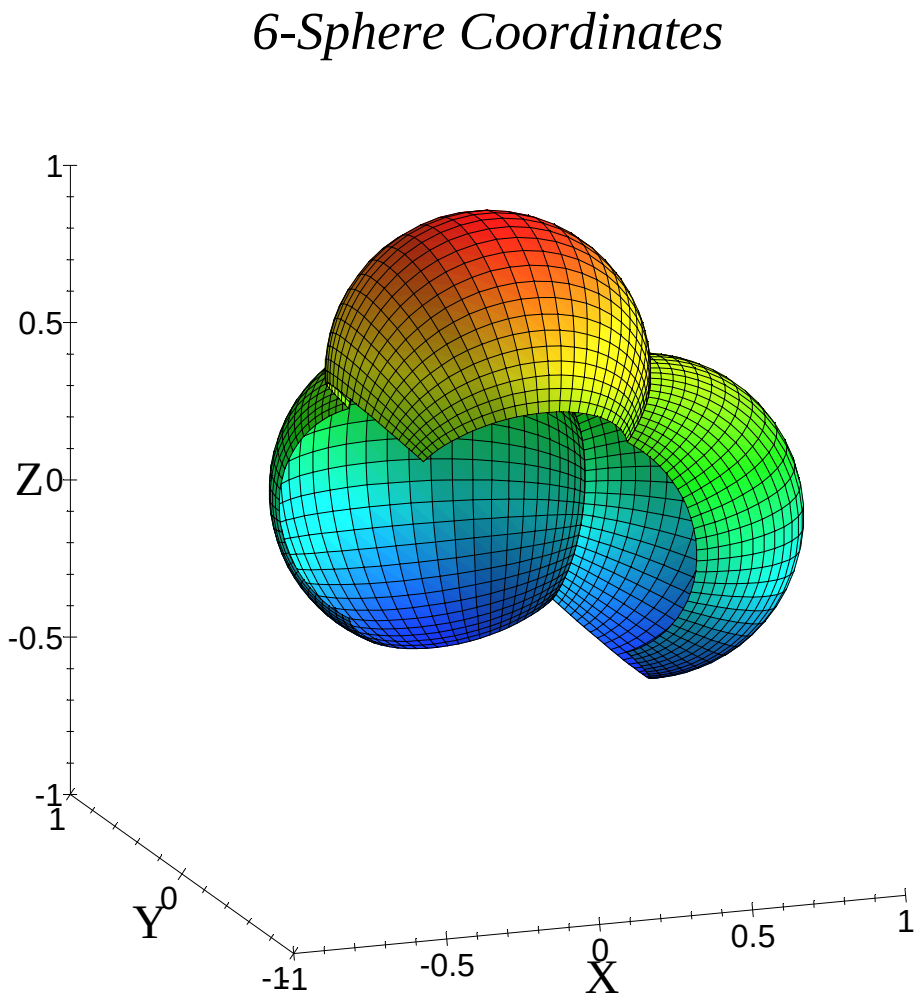
6-Sphere

```
sixsphere:                    (6-sphere)
  x = u/(u^2+v^2+w^2)
  y = v/(u^2+v^2+w^2)
  z = w/(u^2+v^2+w^2)
```

```
xyz := fn( [ u/(u^2+v^2+w^2),
             v/(u^2+v^2+w^2),
             w/(u^2+v^2+w^2) ], u,v,w );
```

$$xyz := (u, v, w) \rightarrow \left[\frac{u}{u^2+v^2+w^2}, \frac{v}{u^2+v^2+w^2}, \frac{w}{u^2+v^2+w^2}\right]$$

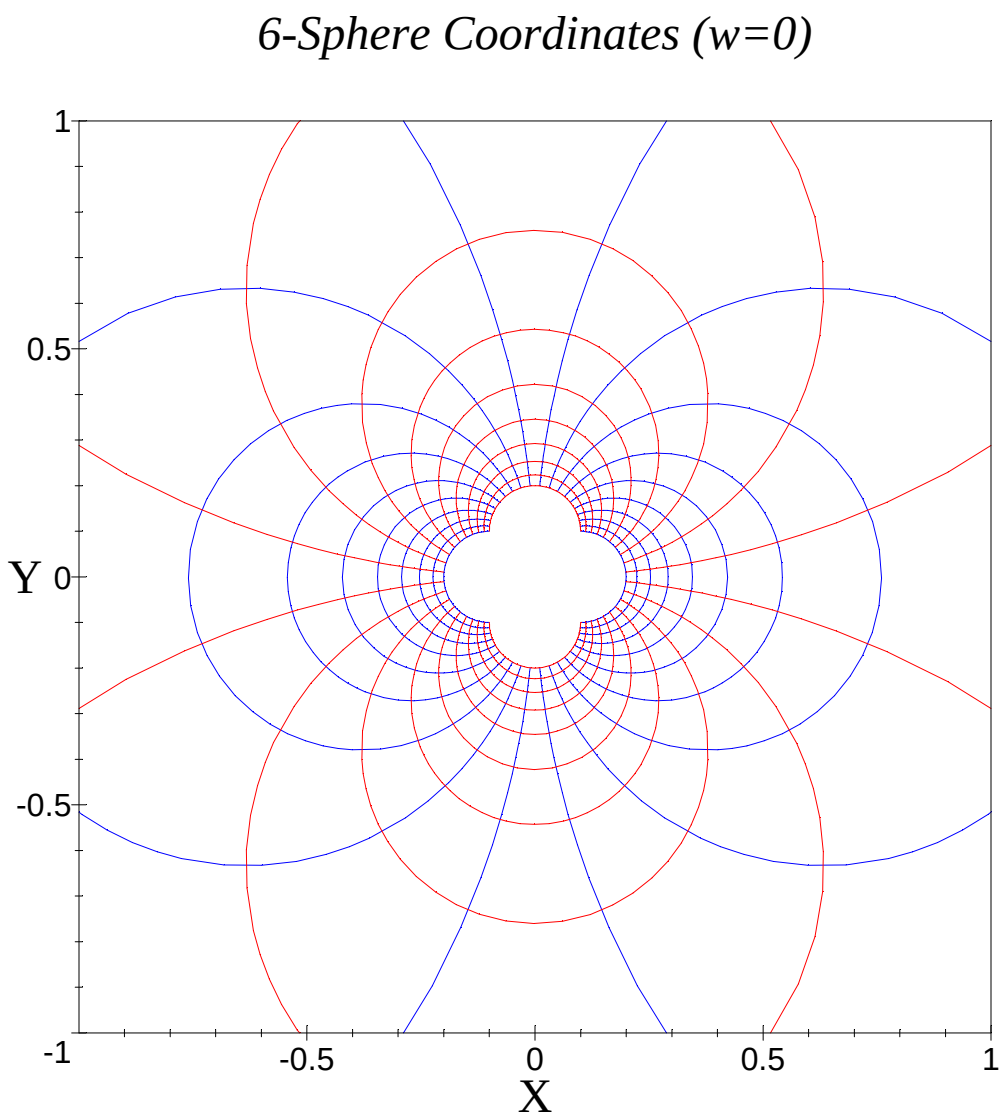
```
coordplot3D( xyz, 1, 1, 1, -1..1, -1..1, -1..1,
             [-110,75], [-1..1, -1..1, -1..1],
            `6-Sphere Coordinates` );
```




```
xy := fn( [u/(u^2+v^2),
           v/(u^2+v^2)], u, v );
```

$$xy := (u, v) \rightarrow \left[\frac{u}{u^2 + v^2}, \frac{v}{u^2 + v^2}\right]$$

```
coordplot2D( xy, -5..5, -5..5,
             [-1..1,-1..1], 20,
            `6-Sphere Coordinates (w=0)`,
             labels=[`X`,`Y`] );
```



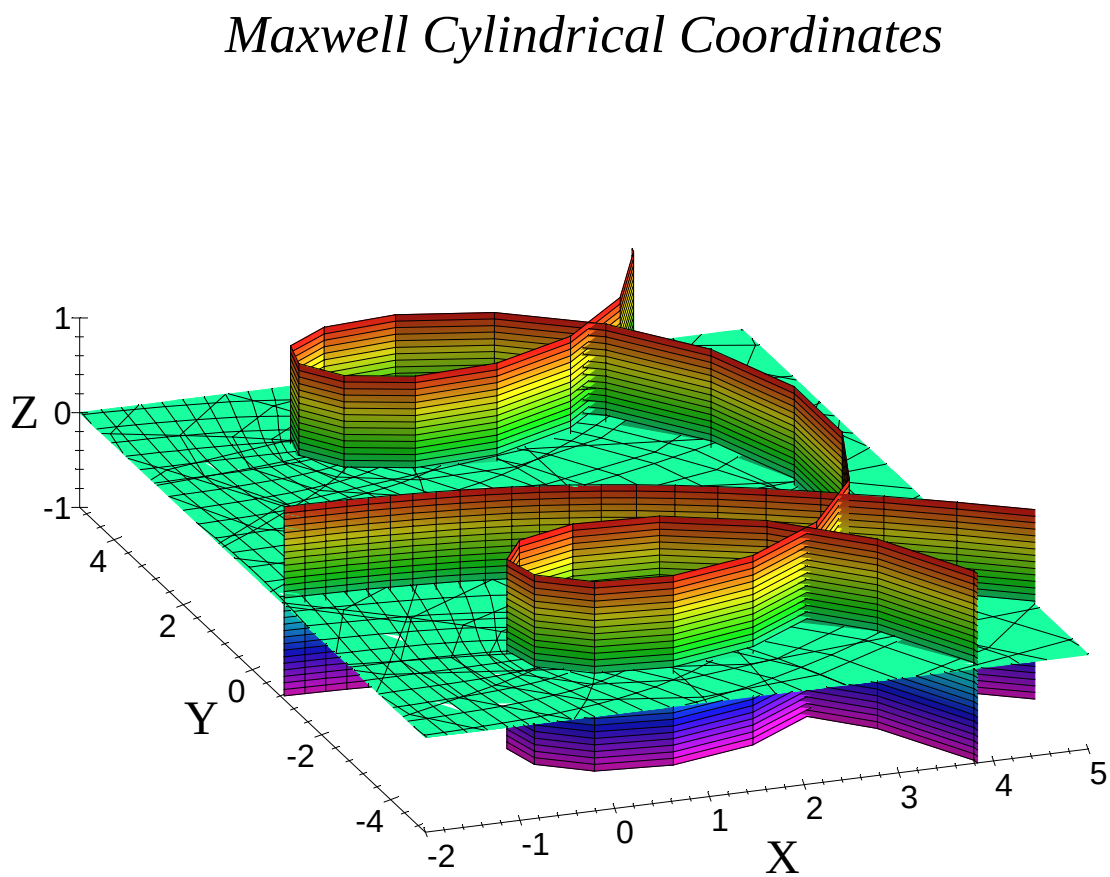
Maxwell Cylindrical

```
maxwellcylindrical:
  x = a/Pi*(u+1+exp(u)*cos(v))
  y = a/Pi*(v+exp(u)*sin(v))
  z = w
```

```
xyz := fn( [ (u+1+exp(u)*cos(v)),
             (v+exp(u)*sin(v)), w ], u,v,w );
```

$$xyz := (u, v, w) \rightarrow [u + 1 + \mathbf{e}^{u} \cos(v), v + \mathbf{e}^{u} \sin(v), w]$$

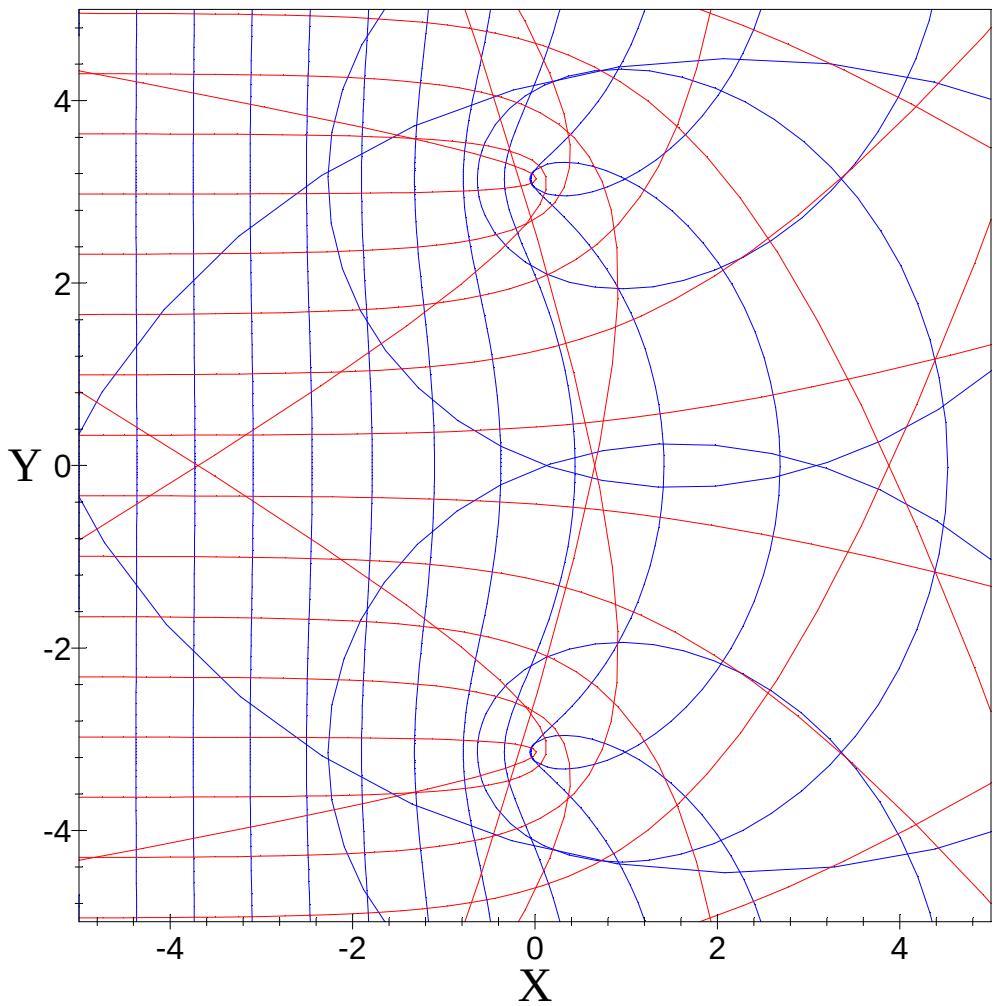
```
coordplot3D( xyz, 0.8, -Pi/4, 0, -3..3, -2*Pi..2*Pi, -1..1,
             [-110,70], [-2..5, -5..5, -1..1],
            `Maxwell Cylindrical Coordinates` );
```




```
xy := fn( [(u+1+exp(u)*cos(v)),
           (v+exp(u)*sin(v))], u, v );
```

$$xy := (u, v) \rightarrow [u + 1 + \mathbf{e}^{u} \cos(v), v + \mathbf{e}^{u} \sin(v)]$$

```
coordplot2D( xy, -6..6, -2*Pi..2*Pi,
             [-5..5,-5..5], 20,
```

```
`Maxwell Cylindrical Coordinates (w=0)`,
 labels=[`X`,`Y`] );
```

Maxwell Cylindrical Coordinates (w=0)