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Updating the Magnitudes of the Planets in *The Astronomical Almanac*

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The content of this Tech Note has been superseded by Hilton, J. 2005, AJ, 129, 2902. However, the tech note may be of interest because it documents the method of computing the magnitudes of Mercury and Venus used in the *AsA* 2005 and 2006.

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Technical Note

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ABSTRACT

Currently, the magnitudes of the planets in the *Astronomical Almanac* are based on the work of *Harris* (1961). It is the intention of this technical note to analyze whether or not Harris's values for the magnitudes of the planets are still the ones that should be presented in the *Astronomical Almanac*. If more up to date values exist, then the aim here is to make a recommendation as to which value should be used in the *Astronomical Almanac*.

INTRODUCTION

The apparent visual magnitude of a planet, V, is given by the equation

$$V = V(1,0) + 5\log_{10}(rd) + \Delta m(i)$$

where $V(1,0)$ is the magnitude of the planet as seen from 1 AU and a phase angle of 0°, r is distance of the planet from the Sun, d is the distance from the Earth to the planet in AU, and $\Delta m(i)$ is the correction for the phase angle i (Hilton, 1992).

Strictly speaking, the quantity $\Delta m(i)$ or *phase coefficient* is measured empirically. In practice, the phase coefficient is determined from a polynomial relation whose coefficients are determined from observations of the planets. Except for the inferior planets, a linear relation is assumed to be sufficient to determine the phase coefficient.

Since at least the 1984 edition of the *Astronomical Almanac*, the phase coefficients used for the planets are those presented in Harris (1961). Harris did not determine the coefficients but was reporting on the work of others. For example, the phase coefficients of Mercury and Venus in Harris were determined by Danjon (1949).

The source for the values used for $V(1,0)$ is unknown. the values are very similar to those presented in Harris, but some values differ.

The object of this technical note is to evaluate Harris in light of other more recent research on the apparent magnitudes of the planets to determine what values should be used for $V(1,0)$ and the polynomial degree and coefficients for the phase coefficients that will best serve that *Astronomical Almanac*.

MERCURY

The phase coefficient currently being used for Mercury is based on Danjon (1949) and corrected by Danjon (1950) is given by:

$$V(1,0) + \Delta m(i) = -0.36 + 3.80(i/100) - 2.73(i/100)^2 + 2.00(i/100)^3$$

Danjon determined this relation from 225 observations of Mercury made between October 15, 1937 and May 22, 1948 over a phase angle of $3° < i < 123°$. The more recent work by Irvine et al. (1968a) includes 31 observations of Mercury in the V band made between June 15, 1963 and May

17, 1965 covering phase angles from 58 ° to 115 °. This work substantially agrees with the Danjon relation for the phase coefficient for Mercury.

Danjon published his data along with his determination of the phase coefficient, but had not described the method he had used to reduce his original data nor included an estimate of the errors in the phase coefficient. Thus I decided to re-reduce the data to determine the polynomial that would best minimize both the differences between the observed and calculated values of the magnitude of Mercury as a function of phase and the uncertainty in the coefficients of the phase coefficient. I used a standard least squares method to determine the values of the coefficients of the polynomial of the phase coefficient. The uncertainty in the polynomial coefficients were determined from the covariance matrix and the standard deviation of the (O - C)s. Danjon determined the photographic magnitude for Mercury. de Vaucouleurs (1964) determined the difference between the photographic and visual magnitude to be –0.17 magnitudes. Thus the best fit for the phase coefficient was found to be

$$V(1,0) + \Delta m(i) = -0.37 \pm 0.02 + 2.12 \pm 0.09(i/100) + 0.81 \pm 0.06(i/100)^2$$

The standard deviation in the (O - C)s was 0.12 magnitudes. Increasing the degree of the polynomial for the phase coefficient did not significantly reduce the standard deviation of the (O - C)s but did greatly increase the uncertainty in the coefficients of the polynomial.

Notice that $V(1,0)$ as well as $\Delta m(i)$ were parameters determined in the least squares solution. Since $V(1,0)$ is not an observable quantity, it becomes another parameter in the least squares fit and the value is subject to change to fit the observed values of planetary magnitude.

The difference between the root mean square deviation of the above quadratic solution for the phase effect of Mercury and a trial cubic solution was insignificant. However, the uncertainty in the parameters in the solution were much greater in the cubic solution than in the quadratic solution. Thus the quadratic solution was chosen as the best fit estimate of the magnitude of Venus rather than using the traditional quadratic fit.

There is one other issue for Mercury. There exists a 3:2 spin-orbit resonance between its rotation and its orbital motion. As a result, the part of Mercury seen at eastern elongation each present as seen from the Earth is always the same. Similarly, every western elongation always presents the same area of Mercury, but it is a different are from that seen at eastern elongation. Thus it is possible that albedo markings on the surface of Mercury might cause the brightness of it to be different between eastern and western elongations. Thus the observational data was broken into eastern and western subsets to test if there were significant differences between the eastern and western elongations. These data were subject to the same least squares analysis and no significant difference was found between the different elongations.

VENUS

Danjon (1949) also determined the magnitude of Venus. He made 335 observations of Venus between October 3, 1937 and September 15, 1947. The value he gave for the phase coefficient was

$$V(1,0) + \Delta m(i) = -4.29 + 0.09(i/100) + 2.39(i/100)^2 - 0.65(i/100)^3$$

where the range of phase observed by Danjon was $0.^\circ9 \leq i \leq 170.^\circ7$.

Since Danjon, there have been two major studies of Venus' brightness as a function of magnitude: Knuckles et al. (1961) and Irvine et al. (1968a).

Knuckles et al. made 56 observations of Venus between June 4, 1954 and October 20, 1960 covering phase angles from 16° to 174°. They then determined the phase coefficient by drawing by

eye what they determined to be the best fit line through the observations and tabulating the result. The tabulated line gives values for the phase coefficient similar to those of from Danjon's algorithm. However, the mean magnitude determined from Knuckles et al. is -0.10 brighter than Danjon.

Irvine et al. (1968a) made 78 observations from May 1963 through Dec. 1965 covering phase angles from 31°.5 through 158°.7. The observed magnitudes generally agreed with Danjon between 35 and about 80°. This is also the portion of the phase-magnitude diagram where Danjon and Knuckles et al. are in closest agreement. At phase angles between 80° and 120°, the Irvine et al. observations generally agree with the Knuckles et al. phase-magnitude curve (about 0.5 mag. less than Danjon). And at phase angles greater than 120°, the Irvine et al. observations generally fall between the Danjon and Knuckles et al. phase-magnitude curves. This is the portion of the phase-magnitude diagram where Danjon and Knuckles et al. show the worst agreement.

As with Mercury, Danjon published his observations for the magnitude of Venus. Thus I re-reduced these observations as well to determine both the best parameters for a least squares fit to the data and estimate the uncertainty in the parameters of the fit. The best fit to the observations, including de Vaucouleurs (1964) correction of –0.13 from photographic to V magnitude is:

$$V(1,0) + \Delta m(i) = -4.35 \pm 0.00 + 0.97 \pm 0.01(i/100) + 0.86 \pm 0.00(i/100)^2$$

The standard deviation in the (O - C)s was 0.05 magnitudes. As with Mercury a quadratic polynomial was found to provide a better fit than the cubic fit determined by Danjon. Note that the V(1, 0) magnitude is 0.06 less than recommended by Harris from Danjon's work. This change concurs with the values for the magnitude of Venus determined by Knuckles et al. and Irvine et al. The scatter in Venus' magnitude is not surprising considering the albedo markings found in the atmosphere discussed by Dollfus et al. (1975).

MARS

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