NEW TECHNOLOGY FOR CELESTIAL NAVIGATION

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Celestial Navigation in the Era of GPS

Just a month ago I attended the annual technical meeting of the Institute of Navigation in San Diego. Almost all of the papers presented there were about current and future applications of the Global Positioning System (GPS); the meeting was an inspirational gathering of the GPS faithful. It is not much of an exaggeration to say that today navigation is virtually synonymous with GPS. This is a development of the present decade, which has seen the completion of the GPS satellite constellation, the shutdown of other electronic means of navigation, and a drastic reduction in the prices of GPS receivers. For Department of Defense vehicles, GPS is the principal means of navigation. U.S. Navy and Marine Corps navigation policy states, "NAVSTAR Global Positioning System (GPS) is the primary external reference system for naval operations requiring POS/NAV and time data."[1]

Yet GPS has operational characteristics and vulnerabilities (including jamming) that may render it unusable or unreliable under certain conditions. Much work is being devoted to developing strategies for GPS outages. Operational plans now must include the contingency that GPS will not be available at the most critical times — a somewhat ironic situation for DoD, which has spent (and continues to spend) billions of dollars on the system. Perhaps anticipating an over-reliance on a single type of "black box" navigation, Navy navigation policy also states, "Every platform/user with a validated requirement shall have a primary and at least one alternate means of position determination. The alternate means must be independent of the primary." [2]

Unfortunately, alternative electronic navigation systems such as Omega and TRANSIT have been decommissioned, and long-term operational support for others, such as LORAN and VOR/DME, is not guaranteed; in any event, the latter are not available worldwide. Some kind of alternative to GPS is needed to comply with Navy policy and provide prudent redundancy for navigation systems. Inertial navigation systems, which are now common on Navy ships and aircraft, are being viewed as the answer. However, there is a complication. These systems are really only a very accurate form of dead reckoning, and they require periodic alignment to some sort of external reference system. That external system could be GPS, of course, but such a mode of operation does not provide a secondary means of navigation that is "independent of the primary."

The stellar reference frame is an alternative to GPS that could be used to align inertial navigation systems. After all, the stars define the most fundamental and accurate inertial system available. As we will see, combining celestial and inertial navigation is not a new idea. Of course, on or near the Earth's surface, a fundamental obstacle to celestial observations is cloud cover: a run of bad weather can separate star sights by a day or more. But an inertial navigation system provides an excellent bad-weather "flywheel" that can carry the stellar fix forward until new observations can be obtained. There is more to be said about the advantages of the celestial-inertial combination, and we will return to the topic later.

Celestial navigation is practiced on a daily basis on Navy vessels. Standard Navy practice relies on quartermasters skilled in the use of hand-held marine sextants and paper-and-pencil sight reduction techniques. The basic method has not changed much in a hundred years, although almanacs and other sight-reduction tools have become more convenient to use. Observations are limited to a few Sun sights during the day and a few star sights during twilight. Because observations with hand-held sextants have typical uncertainties of about one arcminute, celestial fixes are rarely more accurate than several nautical miles. This kind of celestial navigation may be good for "sanity checks" on GPS fixes, and may be useful in an emergency, but its accuracy and availability fall short of many current military requirements.

If celestial navigation is to assume a broader role in the modern Navy's high-tech environment, its limitations will have to be addressed: low accuracy (a few miles), limited time window for observations (horizon must be visible), and low data rate. The sparse amount of celestial data collected over the course of a day results from the use of a human (with other duties) as a detector and computer, the small number of target objects (usually just the Sun and bright stars), and restrictions on the sky area used (altitudes 15° to 65°). It turns out that all of these limitations are a consequence of the way in which celestial navigation is now carried out, rather than being fundamental to the technique. They are a result of the human-intensive observing and computing procedure we use, and in that sense are self-imposed. However, if we are willing to think a bit more broadly about how celestial navigation could be performed, we find that these problems have technical solutions. In fact, as we shall see, most of the needed solutions are available "off the shelf."

Significant improvement to celestial navigation's accuracy and availability will require changes in both the observational hardware and the computational procedure used to obtain a fix. Let us look at the mathematical situation first.

A Child's Garden of Navigation Algorithms — And the Weeds

The calculations that are required for the reduction of a celestial sight, if performed by hand, are slow and error-prone, and discourage the human navigator from taking sights — more tedious work to do! The traditional procedure imposes several other not-so-obvious limitations on the observations. For example, because observations of the Moon and planets require a parallax correction, many navigators avoid these objects, despite the fact that in marginal conditions they may be the only ones visible. Because the Moon is so seldom used, the possibility of Sun-Moon fixes is effectively precluded. All of this argues, if an argument is needed, for a computer program to do the calculations. There are many on the market, some embedded in special-purpose navigational calculators. Any reasonably accurate algorithm, implemented in a user-friendly program, would encourage navigators to broaden their observational habits and obtain more sights.

Beyond this common-sense recommendation for automation of the calculations, it becomes necessary to consider the specific algorithms used. A wide variety of algorithms for celestial navigation are available in the literature. Within the last three decades, in particular, many papers on this subject have been published, the authors motivated by the availability of inexpensive computing power compact enough for even small boats. Some very innovative mathematical approaches to celestial navigation were formulated, and some of these schemes found their way into commercial software products. There are now perhaps a dozen exact solutions of a two-body fix (although I doubt whether these are all mathematically independent). Of course, no prudent navigator would rely on a fix using only two observations (unless no others were available) and these exact solutions are not readily extensible to the more common case of three or four sights. When there are more than two observations, the problem is overdetermined and least-squares techniques can be used. Several least-squares approaches to a multi-star fix have been published. One, by deWit,[3] is based on the plane geometry and straight lines formed by celestial lines of position near the estimated position, a direct mathematical translation of chart-based navigation. It was developed independently by our colleagues at Her Majesty's Nautical Almanac Office and is printed in the back of *The Nautical Almanac* and in the HMNAO publication *Compact Data for Navigation and Astronomy.*[4] In fact, *Compact Data* now includes a PC diskette with software that implements it. The scheme is quite easy to understand and is very robust. Use of plane geometry is an approximation, of course, but the method is quite adequate for the accuracy of ordinary sextant observations. A later least-squares formulation, by Severance,[5] is more mathematically straightforward in that it does not rely on a special geometric construction.

Perhaps the most elegant solution to the multi-body fix problem was published by Paul Janiczek of the (U.S.) Nautical Almanac Office in 1978. It is a vector-matrix approach that fits on one page.[6] An extension of this method, which uses a Lagrange multiplier for normalization, was published in 1991 by Thomas and Frederic Metcalf.[7]

Thus, in 1993, when the Chief of Naval Operations (N6) gave the Naval Observatory the task of providing standard celestial navigation software for Navy fleet use — the STELLA project — we apparently had many choices for the basic algorithm. (And I have not given here a complete survey of all the possibilities.) Initially we were leaning toward use of the Metcalf & Metcalf algorithm. One of the aspects of the project that I got interested in was how to deal with the motion of the ship during the time that a round of sights was taken; we wanted STELLA to handle a "running fix" as rigorously as possible. As it turned out, consideration of this apparently small piece of the overall problem led me to devise a completely different formulation of celestial navigation, one that is now incorporated into STELLA.

I discovered that despite the wide variety in the previously published algorithms, the fundamental developments for all of them assumed two or more co-located observations, something that requires either a stationary observer or simultaneous sights. Neither, of course, is a realistic scenario. In the real world, the observer's position changes during the finite time required to make the observations, so use of any of these algorithms requires transforming a moving-observer problem to a fixed-observer problem. One frequently used procedure is the addition of a motion-of-observer correction to an observed altitude; another is advancing the observation's line of position on the plotting chart. The most important weakness of such procedures is well known: they require data on the motion of the observer's ship over bottom (that is, in latitude and longitude), and the course and speed values used may not be accurate. The accuracy of these quantities is usually limited by our inexact knowledge of the local current. The errors involved are such that, for sights made with ordinary hand-held sextants, difficulties may arise for observations spread over more than about a half-hour. Of course, if the accuracy of the observations could be significantly improved, then an observing period of only a few minutes would become problematic. The possibility of better observational material was something we wanted STELLA to be able to handle. Fortunately, the observations themselves contain information on the actual track of the vessel, so it should be possible to make the sight-reduction procedure self-correcting. In principle, given enough observations, suitably distributed in time and azimuth, we should be able to obtain an estimate of the average over-bottom track of the vessel as part of the solution for the fix. In 1995 I published a development of celestial navigation that incorporates a moving observer as part of its basic construction.[8] This approach correctly represents the propagation of positional error along the observer's track, considered to be a standard rhumb line (loxodrome) traversed at constant speed.[9] Furthermore, the procedure allows, under certain conditions, recovery of information on the vessel's actual course and speed from the observations. This new algorithm, described briefly below, includes the observer's motion as an essential part of the mathematics of celestial navigation, rather than as an add-on. Additionally, because the algorithm is not based on lines of position, it does not preclude observations very close to the zenith, if the instrumentation allows.

Celestial Navigation as an Orbit Correction Problem

Suppose we are given a series of observations taken over an extended period of time from a moving vessel. Is there a way to mathematically develop celestial navigation that includes the vessel's motion in the problem from the outset? Further, can such a development allow us to exploit the observations to correct our initial estimates of the course and speed of the vessel, as well as to provide a fix for a given time?

Our problem is quite similar to "orbit correction" problems faced by astronomers who deal with the dynamics of solar system bodies. (See Figures 1 and 2.) Given a series of observations of some moving object in the solar system — an artificial satellite, a deep-space probe, an asteroid, or a planet — we want to be able to compute the position of the object at

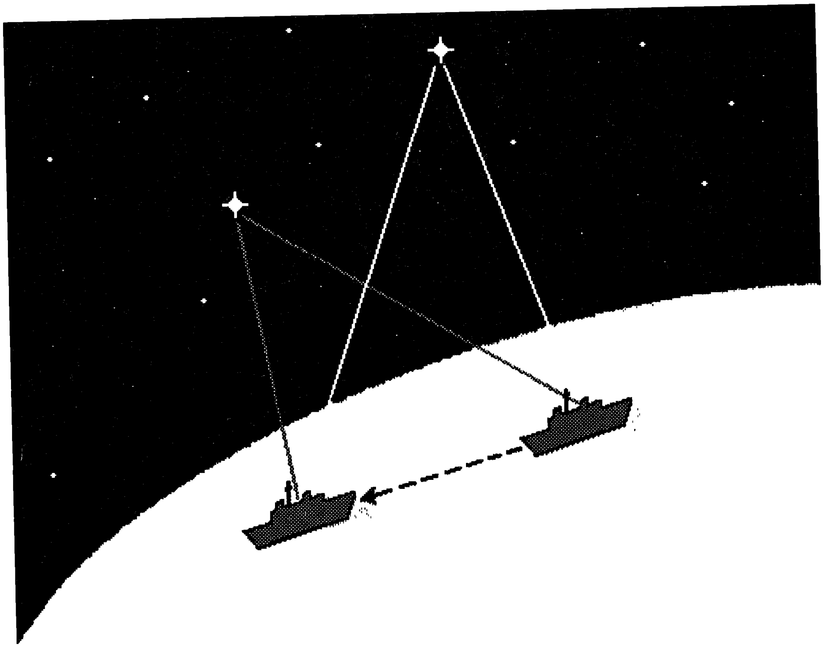

Figure 1. Navigation problem: moving observer, fixed celestial object(s).

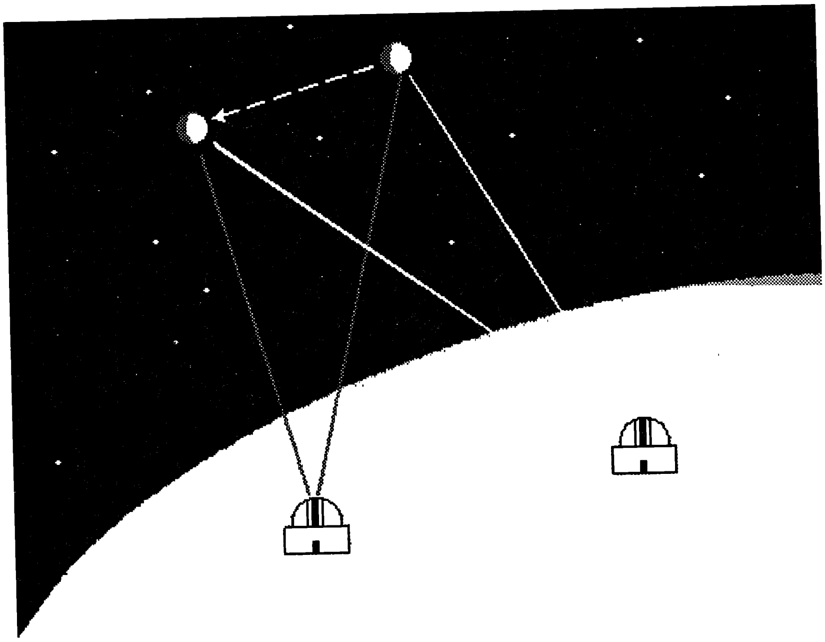

Figure 2. Astronomical problem: moving celestial object, fixed observer(s).

any given time. We know the laws of motion that the body obeys, but there is an infinite set of physically possible trajectories. Therefore, we must use the observations to determine the initial conditions, or orbital elements. The orbital elements are six parameters that specify the object's position and motion, in three dimensions, at a designated time. Once these six parameters have been determined, the object's position at any other time can be computed. The problem is the same regardless of what kind of observations are available. The observations may consist of simultaneous measurements of both celestial coordinates, or the observations may be only of range (distance). In the latter case we use a series of one-dimensional observations to solve a six-dimensional problem. As long as we have at least six observations (suitably distributed) the problem is solvable.

The running fix problem in celestial navigation is analogous. The moving object of interest is the observer's ship. The fact that the observations are taken *from* the moving object rather than *of* the moving object does not change the nature of the problem. However, in celestial navigation, the problem is four-dimensional rather than six-dimensional because ships are constrained to move over the two-dimensional surface of the Earth. The sailing formulas for rhumb-line tracks are the "laws of motion." We have a series of one-dimensional observations — sextant altitudes — from which we wish to determine, for a given time, the four "orbital elements" of the vessel: latitude, longitude, course, and speed. Once these have been determined, the vessel's position at any other time can be computed. As long as we have four or more observations, well distributed in azimuth and time, the problem is solvable.

The orbit correction problem is usually dealt with through a process called differential correction, which uses linearized equations in a least-squares formalism. This requires that we have some initial knowledge of the trajectory of the object of interest, which is almost always the case in both astronomy and navigation. This allows us to make a reasonably accurate estimate of the value of any observed quantity (e.g., declination or altitude) for any given time. The small difference between the observed value and the computed estimate can be accounted for by corrections to selected parameters in our *a priori* model of the object's motion.

Of course, a ship does not follow an exact rhumb-line course at constant speed, but is subject to random variations in wind, current, and steering. A vessel's path over bottom is a somewhat irregular line. The method of least squares, applied to this problem, assumes that the ship's excursions from a rhumb line have a normal (Gaussian) distribution, even though that is unlikely to be rigorously true. Given a sufficient number of observations, the algorithm yields the parameters for a kind of average rhumb-line track over bottom, which is, presumably, what is desired. More problematic are systematic changes in the current or wind that occur over time scales of hours. In such circumstances the ship's track may not be well represented by a single constant-speed rhumb line. However, if the ship's track can be modeled as a series of connected rhumb lines, then a generalization of the algorithm can be used.[10] The generalization, which is included in STELLA, allows observations taken over multiple voyage legs to be combined into a single solution.

The algorithm has been extensively tested using artificially generated data, both with and without random errors. Many examples found in navigation texts have been reduced again using it, and the results compared with other sight-reduction algorithms. Additionally, because STELLA was tested before release on board deployed Navy vessels, the method was checked in real-world applications. The algorithm works well and is robust. Statistical correlations among the parameters being solved for are usually low. The tests with perfect artificial observations have demonstrated the mathematical correctness of the algorithm.

However, the full power of the procedure has probably not been used so far in practice. Consider the traditional round of sights, in which a small number of observations (usually three to five) are taken within a short period of time, in twilight, and reduced to determine a fix. The uncertainties of hand-held sextant observations from a moving ship are such that almost any sight-reduction procedure is adequate for this case, and the algorithm described above does not have significant practical advantages over others. Course and speed corrections cannot be determined with such a limited observation set, and STELLA will not attempt to do so. For such cases, all reasonable algorithms give essentially the same answer.

The advantages of this new algorithm become evident when navigational practice is extended beyond the usual twilight round of sights or noon Sun line. Even with ordinary hand-held marine sextants, more flexibility in navigational procedures is possible than is usually practiced. For example, at high latitudes, long periods of twilight allow for extended sets of observations. Sun-Moon fixes are geometrically possible during about half of all days. Sun or Moon observations from early or late in the day can be combined with twilight observations. Observations of the stars and planets are possible at night near full Moon when the sky is bright enough to make the horizon visible. But exploiting the full advantages of the algorithm would probably require new hardware, for example, an automated star tracker with an artificial horizon that could observe all night, or, in the near infrared, during the day. This leads us to consider the prospects for applying new hardware technology to the task of taking celestial observations.

Improving the Observational Data

Before we consider some of the new hardware possibilities, we should be clear on what it is we need to measure. To obtain latitude and longitude using observations of stars, which are for practical purposes infinitely distant, the essential measurement is the angle between the star and the local gravity vector at an accurately known time. The determination of time at sea has, of course, an interesting history in itself, but for present purposes I consider precise timekeeping to be a solved problem.

The gravity vector is indicated by a plumb bob, liquid surface, or floating bubble. Aircraft sextants for many years used a bubble in the field of view to indicate the vertical direction. For a standard marine sextant, the horizon, which we assume is a circle orthogonal to the gravity vector through the observer, provides a surrogate for a vertical reference on the instrument. This use of the sea horizon, the tangent to a liquid surface external to the vessel, has advantages that will soon become clear. It is interesting to note that if celestial objects were sufficiently close, the observer's position could be obtained without a determination of the local vertical — triangulation, similar to that used for conventional aids to navigation, could be used. The Moon is almost close enough for this (measuring the position of the Moon against the star background to one arcsecond would yield position on Earth to about one mile) but artificial Earth satellites would work much better (in principle, at least). However, for conventional stellar navigation, a gravity reference is needed.

Each observation ("sight") from a marine sextant consists of a measurement of the altitude of a celestial body above the visible horizon. There can be no dispute that the sextant is an extraordinarily successful instrument for its task. It is remarkable that the basic design of the marine sextant has not changed since the 18th century, when sextants (actually octants) replaced the cross-staff and back-staff. Over the past two hundred years, countless vessels of all sizes have sailed to all parts of the world using only a sextant for offshore fixes.

Occasionally there are initiatives to improve the sextant. The Nautical Almanac Office was involved in several such projects.[11,12] Improvements included digital encoders to read out the angles, image intensifiers, and direct connection to a computer, which kept track of time. The most recent of such projects resulted in a prototype "automatic sextant" connected to a small calculator programmed to reduce the sights. Apparently the Navy did not choose to follow up on these developments. More recently, some commercial sextants have come equipped with modern night vision devices that have received favorable reviews. The night vision addition allows the horizon to be seen when it would otherwise be invisible. It's easy to imagine other possible improvements, such as automatic averaging of measurements or some form of image stabilization.

However, improvements to the sextant are unlikely to change the basic paradigm of shipboard celestial navigation, because the task would remain human-intensive. In contrast, most modern astronomical instrumentation is designed to remove humans from the observing process as much as possible, as a way of improving the efficiency of large telescopes and other expensive equipment. Such instrumentation, which could improve both the number and accuracy of observations made for celestial navigation, has not been exploited for shipboard use. However, some very advanced technology has been used for a related application — space navigation — and the same kinds of devices can, I believe, be profitably applied to surface and air navigation. A not unreasonable expectation for this technology is the acquisition of large numbers of star altitudes, day or night, at an accuracy approaching one arcsecond, equivalent to 31 meters on the surface of the Earth. This is comparable to GPS standard positioning accuracy.

Since the early days of the space age, automated celestial observing systems have been used on missiles, satellites, and planetary exploration spacecraft as an aid to navigation. Strategic missile systems such as Polaris, Poseidon, Trident, and MX have used compact star trackers in the powered phase of flight to determine the absolute orientation of the vehicle for the inertial guidance system. The more modern of these units achieve sub-arcsecond angular precision, a fact that has motivated some of the star catalog work done at the Naval Observatory over the past several decades. Many satellites use star sensors to determine attitude. The Space Shuttle has automated star trackers in its nose. Deep space missions may use star or Sun sensors en route for attitude determination, and science camera images of the target body against the star background as part of the terminal navigation program. Star trackers have evolved from single-star to multi-star capability. Thus, space systems provide a substantial technological base in the automated measurement of stellar angles.

An example of a state-of-the-art star tracker is Lockheed's AST-201 system.[13] Using what amounts to a standard camera lens with a charge coupled device (CCD) array in its focal plane, this unit can detect stars down to visual magnitude 7, the exact limiting magnitude depending on the unit's rotation rate. The star tracker contains its own star catalog and star pattern recognition software, and is designed to operate as a "black box" that receives stellar photons as input and provides digitized orientation angles as output. The orientation accuracy is several arcseconds about axes parallel to the focal plane. The unit is approximately 15 cm × 15 cm × 30 cm, including the lens shade, weighs about 4 kg, and is, of course, space qualified.

Would an automated star tracker be practical for surface or air navigation? In the late 1980s, Northrop designed a system called the Optical Wide-angle Lens Startracker (OWLS) that it packaged with an aircraft inertial navigation system.[14] Using a holographic lens that could simultaneously image three 3° fields of view, each with its own focal plane detector array, the OWLS could deliver arcsecond-level orientation angles to the INS. The OWLS operated in the far red (R band, λ 0.6-0.8 μm) so that it could detect stars down to R magnitude 5 at sea level in daylight. Clearly Northrop thought its system had broad application: "...astro-inertial navigation offers a practical solution for high-precision, autonomous navigation for surface ships, commercial aircraft, cruise missiles, strategic aircraft, remote piloted vehicles, and hypersonic vehicles."[15] Although the system apparently never achieved such widespread use, its documentation presents a very clear picture of the possibilities.

As we have seen, compact, self-contained instrumentation is available for automated determination of star position angles. However, we have not yet discussed the other measurement required for latitude-longitude fixes: a determination of the local vertical. That leads us to again consider the role of inertial navigation systems.

Which Way is Up?

Determining the exact direction of the local gravity vector seems at first thought to be a trivial task. The measurement is fairly straightforward for a fixed location. Modern tiltmeters or accelerometers are sensitive to the direction of gravity to arcsecond (or better) precision. It is true that for accurate position determination with respect to the Earth's reference ellipsoid, the apparent gravity vector must be corrected for "deflection of the vertical." This correction, which can amount to several tens of arcseconds, accounts for small-scale irregularities in the Earth's mass distribution. Fortunately, there are models and maps of the Earth's gravity field that are becoming more detailed and accurate all the time.

Unfortunately, other complications arise for a moving observer. Consider a hypothetical vehicle that is moving smoothly across the surface of the Earth. Assume motion with a constant heading, speed, and altitude, with negligible motion-related accelerations (aside from Coriolis forces, which are generally small and easily computable). In such a case, the gravity vector could be measured directly with any of the standard instruments. Using the STELLA algorithms, a series of measurements of the angles between the local gravity vector and an ensemble of stars could provide an autonomous determination of location at a given instant, as well as course and speed.

Of course, our hypothetical smoothly moving vehicle represents a rather rare, if not nonexistent, case. In real-world conditions, a moving vehicle is subject to a variety of accelerations from both internal and external sources. These accelerations cannot in principle be separated from that due to the Earth's gravity, so that any instantaneous measurement of the local gravity vector from inboard devices, such as tiltmeters or accelerometers, is highly contaminated. We can now understand why, for a sextant user, the sea horizon works better than a direct measurement of the local vertical: the horizon is not subject to the accelerations of the ship.

The problem of determining the true local vertical from a moving vehicle leads us back to inertial navigation systems, which have become ubiquitous on aircraft, missiles, and ships. As previously noted, these units can be thought of as an automated form of very precise dead reckoning. Each system combines a set of gyros, a set of accelerometers, and a computer. The unit must be initialized when the vehicle is at a known location. Using a continuous, rapid series of gyro and accelerometer measurements, the INS can compute the vehicle's instantaneous position and velocity at any later time. The system is thus self-contained after initialization. The accuracy of these systems varies widely, depending on size, cost, and acceleration environment. Typical specifications for aircraft

INS call for drifts within one nautical mile per hour of operation, but ship INS specifications are one to two orders of magnitude better.

As part of its navigation calculation, an inertial navigation system must *infer* the direction of the local vertical at each computation step. Due to gyro drift and other errors, this inference may not be as accurate as we would like (errors may accumulate at a rate of an arcsecond to an arcminute per hour), but it is likely to be better than any alternative. Thus, an INS can provide a usable, although not ideal, reference direction for astronomical measurements. Essentially, the INS becomes the plumb bob.

However, the astronomical measurements can be used to help correct certain INS errors — star tracker observations provide a link to an external reference frame that can be used to constrain the INS gyro drift. (The Kalman filter in the INS computer directly uses the star tracker data.) Both orientation and position determinations are significantly improved. And, the INS will continue to provide navigation data (although of lesser accuracy) even if stars cannot be observed because of cloud cover. This kind of tightly coupled celestial-INS system has been most widely used for missile guidance systems, with great success. The combination is not perfect, since it is insensitive to at least one INS error mode (the Schuler oscillation),[14] but it is a proven technology with a substantial engineering base.

Conclusion

Far from being a dying art, celestial navigation is moving into the 21st century as a highly sophisticated technology. Unfortunately, since much of the new hardware has been developed for space systems, many of the technological advances have been invisible to those outside the aerospace engineering community. I believe that much of the work that has gone into star trackers for space applications can be brought down to Earth to serve in new-generation air and sea navigation systems.

In particular, combining automated star trackers with inertial navigation systems seems to be a synergistic match. Inertial and celestial navigation have complementary characteristics. After initialization, INS is completely self-contained and has no coupling to any external reference system; celestial provides a direct link to the most fundamental inertial reference system available. INS units require initial alignment using positioning data from another source; celestial is completely autonomous. INS accuracy degrades with time from initial alignment; celestial fix accuracy is not time dependent. INS units are oblivious to the weather; celestial is highly weather-dependent. Yet, despite their differences, both INS and celestial are passive, jam-proof, and in operational use are not dependent on shore or space components.

Tightly coupled celestial-INS systems have a history of success in certain applications. However, they have not been used on ships, even though modern sensors in the far red or near infrared would allow significant numbers of stars to be observed both night and day at sea level. It remains to be seen what modifications in design might be required for a shipboard environment, and whether these systems could achieve GPS-like accuracy afloat.

The possibility of other celestial-inertial configurations should also be explored. An accurate celestial-only navigation fix obtained without the use of the INS vertical reference would be a great advantage, but not one easily achieved. For example, adding a horizon sensor to a shipboard star tracker would allow for such fixes, but only when the horizon was a distinct line, and then with uncertain accuracy. Another possibility is using artificial satellites observed against the star background to form a navigation solution without a vertical reference. (Optical observation of satellites for navigation is being studied at Draper Lab.) It might even be possible to determine the local vertical from the effects of atmospheric refraction on star observations alone, although large numbers of very precise observations would be required.

When navigation methods are combined, the objective is to use the strengths of one technique to compensate for the weaknesses of another in a way that results in significantly higher accuracy and reliability. To this end, the Navy is in the process of deploying the Navigation Sensor System Interface (NAVSSI),[16] a real-time computer that provides the shipboard navigator with "one stop shopping" for position, velocity, and heading information from GPS, INS, fathometer, gyrocompass, radar, and other sources. The STELLA algorithms are being added to the NAVSSI software, but there are no plans for any kind of star sensor to provide the kind of data the system needs to fully use those algorithms. As we have seen, there is hardware available to provide such data — why not use it?

As our defense forces rely increasingly on GPS, it is important that this dependence does not become a single-point-failure risk for military operations. Independent alternatives to GPS are needed and are required by official policy. Imaginative application of available technology can ensure that celestial navigation has as much of a role to play in the future as it has in the past in helping to provide safe passage for our military forces worldwide.

NOTES

1. Chief of Naval Operations/Commandant of the Marine Corps joint letter Ser 09/1U500942 of 1 August 1991, p. 2

2. Ibid. A new navigation policy letter is being drafted, but the two statements quoted in the text are being retained.

3. C. DeWit, "Optimal Estimation of a Multi-Star Fix," *Navigation*, Journal of the Institute of Navigation, 24 (Spring 1997), 67-71.

4. B.D.Yallop and C.Y. Hohenkerk, *Compact Data for Navigation and Astronomy for the Years 1996-2000* (HMSO, London, 1995).

5. R. W. Severance, "Overdetermined Celestial Fix by Iteration," *Navigation*, Journal of the Institute of Navigation, 36 (Winter 1989-90), 373-378.

6. R. Watkins and P. M. Janiczek, "Sight Reduction with Matrices" (Forum), *Navigation*, Journal of the Institute of Navigation, 25 (Winter 1978-79), 447-448. (Despite the order of the authors' names, it was Janiczek that devised the method.)

7. T. R. Metcalf and F.T. Metcalf, "On the Overdetermined Celestial Fix," *Navigation*, Journal of the Institute of Navigation, 38 (Spring 1991), 79-89.

8. G. H. Kaplan, "Determining the Position and Motion of a Vessel from Celestial Observations," *Navigation*, Journal of the Institute of Navigation, 42 (Winter 1995), 631-648.

9. It was rather surprising that closed-form expressions for latitude and longitude as a function of time, for rhumb lines on the Earth's ellipsoid, were not available in the literature. Thus, deriving accurate sailing formulas was one of the first orders of business for the STELLA project. The formulas used in STELLA are described in G. H. Kaplan, "Practical Sailing Formulas for Rhumb-Line Tracks on an Oblate Earth", *Navigation*, Journal of the Institute of Navigation, 42 (Summer 1995), 312-326.

10. G. H. Kaplan, "A Navigation Solution Involving Changes to Course and Speed," *Navigation*, Journal of the Institute of Navigation, 43 (Winter

1996-97), 469-482.

11. S. Feldman, P. K. Seidelmann, and G. G. Barton, "Advances in Celestial Navigation," *Naval Engineers Journal* (August 1974), 65-76.

12. J. A. Decker, Jr., "Automatic Sextant," *Proceedings of the Forty-First Annual Meeting*, Institute of Navigation (June 1985), 111-117.

13. "AST-201 Star Tracker System Specifications, Rev. D," LMMS-F426359-D, Lockheed Martin Missiles & Space Advanced Technology Center (September 1998).

14. S. Levine, R. Dennis, and K. L. Bachman, "Strapdown Astro-Inertial Navigation Utilizing the Optical Wide-angle Lens Startracker," *Navigation*, Journal of the Institute of Navigation, 37 (Winter 1990-91), 347-362.

15. Ibid., p.362.

16. R. A. Greer, "The Navigation Sensor System Interface Project," *The Journal of Navigation*, 46 (May 1993) , 238-244.